The tuned mass-damper-inerter (TMDI) for passive vibration control of multi-storey building structures subject to earthquake and wind excitations

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ACKNOWLEDGEMENTS

• Prof. Nick Karcanias
  2011:
  Do you know about what the inerter is?
  Can we use it for passive vibration control of buildings?

• Marian Laurentiu
  City University London studentship (2012-2015)
  PhD thesis (viva 12/2016):
  “The tuned mass damper inerter for vibration suppression and energy harvesting in dynamically excited structural systems”

• EPSRC
  Big Pitch- Bright Ideas award (10/2014)
  *Multi-objective performance-based design of tall buildings using energy harvesting enabled tuned mass-damper-inerter (TMDI) devices*

• Dr Francesco Petrini (Performance-based wind engineering)
• Dr Jonathan Salvi (Smart structures and control)
• Assoc. Prof. Alexandros Taflanidis, University of Notre Dame, USA
Presentation Outline

- **Introduction/Motivation**
  - The classical tuned mass-damper (TMD)
  - Motivation for the tuned mass-damper-inerter (TMDI)

- **TMDI for single-degree-of-freedom (SDOF) primary structures**
  - Optimum design formulae
  - Vibration suppression performance and weight reduction

- **TMDI for multi-storey seismically excited structures**
  - Eurocode 8 compatible seismic design and assessment of TMDI equipped building structures
  - Reliability-based optimum design of TMDI equipped building structures accounting for higher modes of vibration

- **TMDI for wind excited tall buildings**
  - Vortex shedding and occupants comfort
  - Parametric analysis for a benchmark wind excited TMDI equipped 74-storey building

- **Concluding remarks**
The classical tuned mass damper (TMD)

For primary structures modelled as single-degree-of-freedom (SDOF) systems

- Additional mass $m_{\text{TMD}}$ attached to the primary structure via a linear spring $k_{\text{TMD}}$ and dashpot $c_{\text{TMD}}$ in parallel.

- Given a primary structure and attached mass, find $k_{\text{TMD}}$ and $c_{\text{TMD}}$ such that the response of the primary structure is minimised.

- The larger the attached mass, the better level of vibration suppression is achieved in terms of peak response along a wider frequency band.
The classical tuned mass damper (TMD)

For primary structures modelled as multi-degree-of-freedom (MDOF) systems

- The $m_{TMD}$ mass is attached to the “lead” mass via a linear spring $k_{TMD}$ and dashpot $c_{TMD}$ in parallel.

- Given a primary structure and attached mass, find $k_{TMD}$ and $c_{TMD}$ to control the first (fundamental) mode shape.

- Wind induced vibrations: $m_{TMD} \approx 0.2\% - 2\%$ of the total building mass

- Earthquake-induced vibrations: $m_{TMD} \gg 2\%$ of the total building mass

![Diagram of a tuned mass damper system]
TMD applications in buildings

Suppression of wind induced vibrations

Ni/Zuo/Kareem (2011)
Zuo/Tang, JIMSS (2013)
Edificio Park Araucano, Santiago, Chile

Suppression of earthquake induced vibrations

http://sirve.cl/archivos/proyectos/diseno-de-sistema-de-proteccion-sismica-edificio-parque-araucano
Protection of buildings for earthquakes
- Conventional design philosophy: allows for inelastic deformations \((a)\)
- Passive vibration control: incorporates different devices to reduce the ground motion induced oscillations \((b)-(d)\)

\[
\begin{align*}
(a) & \\
(b) & \\
(c) & \\
(d) & \end{align*}
\]

- Passive response control systems: \((b)\) seismic isolation, \((c)\) energy dissipation devices, \((d)\) Passive TMDs (linear, easy to design)

BUT: Require the attachment of very large attached masses
Motivation for the tuned mass-damper-inerter (TMDI)

Need for a TMD-based passive dynamic vibration absorber for earthquake engineering applications that:
- does not involve excessively large attached mass
- is linear
- is amenable to the standard TMD optimum design approaches

De Angelis/Perno/Reggio, *EESD* (2012)
The concept of the inerter and its mass amplification effect

- The inerter is a linear device with two terminals free to move independently and develops an internal (resisting) force proportional to the relative acceleration of its terminals (Smith 2002)

\[ F = b(\ddot{u}_1 - \ddot{u}_2) \]

- The constant of proportionality \( b \) ("inertance") is in mass units (kg)

- Single-degree of freedom oscillator connected to the ground via an inerter:

\[ (m_1 + b)\ddot{x}_1 + c_1\dot{x}_1 + k_1x_1 = -m_ia_g \]

- The inerter increases the \( m_1 \) mass:

\[ m_1 \rightarrow m_1 + b \]

"mass amplification effect"

Note: \( F = k(u_1 - u_2) \)

\[ F = c(\ddot{u}_1 - \ddot{u}_2) \]

Smith, IEEE (2002); Takewaki/Murakami/Yoshitomi/Tsuji, SCHM (2012); Chen/Hu/Huang/Chen, JSV (2014)
Flywheel-based inerter with rack and pinion mechanism  Smith, IEEE (2002)
- Assume a mechanical realisation of the inerter comprising a plunger that drives a rotating flywheel through a rack, pinion and gearing system:

\[
b = m_f \frac{\gamma_f^2}{\gamma_{pr}^2} \left( \prod_{i=1}^{n} \frac{r_i^2}{pr_i^2} \right)
\]

- The constant of proportionality \( b \) (inertance) - mass units (>> physical mass of device)
- Inerter can have an inertance ("apparent" mass) orders of magnitude higher than its physical mass.
The tuned mass-damper-inerter (TMDI)

A linear potentially lighter than the TMDI passive control solution with the design simplicity of the TMD. Marian/Giaralis, *PBEE*, (2013, 2014)

**SDOF primary structures**

- Mass attached to the primary structure via a linear spring and a viscous damper + an *inerter* device which connects the attached mass to the ground.

**MDOF primary structures**

- TMDI as an *inter-story connective device placed in a ‘diagonal’ configuration*.

For $b=0$:
- Tuned mass-damper (TMD)

For $m_{TMDI}=0$:
- Tuned inerter-damper (TID)

Equations of motion

Time domain:

\[
\begin{bmatrix}
    m_{TMDI} & b \\
    0 & m_1
\end{bmatrix}
\begin{bmatrix}
    \ddot{x}_{TMDI} \\
    \ddot{x}_1
\end{bmatrix}
+ \begin{bmatrix}
    c_{TMDI} & -c_{TMDI} \\
    -c_{TMDI} & c_1 + c_{TMDI}
\end{bmatrix}
\begin{bmatrix}
    \dot{x}_{TMDI} \\
    \dot{x}_1
\end{bmatrix}
+ \begin{bmatrix}
    k_{TMDI} & -k_{TMDI} \\
    -k_{TMDI} & k_1 + k_{TMDI}
\end{bmatrix}
\begin{bmatrix}
    x_{TMDI} \\
    x_1
\end{bmatrix}
= \begin{bmatrix}
    m_{TMDI} \\
    m_1
\end{bmatrix}
a_g
\]

\[\omega_{TMDI} = \sqrt{\frac{k_{TMDI}}{m_{TMDI} + b}}\]
\[\zeta_{TMDI} = \frac{c_{TMDI}}{2(m_{TMDI} + b)\omega_{TMDI}}\]
\[\mu = \frac{m_{TMDI}}{m_1}\]
\[\nu_{TMDI} = \frac{\omega_{TMDI}}{\omega_1}\]
\[\beta = \frac{b}{m_1}\]

Frequency domain:

\[G_1(\omega) = \frac{x_1}{a_g} = \frac{(1 + \mu)\omega_{TMDI}^2 - \omega^2 + i2\zeta_{TMDI}(1 + \mu)\omega_{TMDI}\omega}{(1 - \frac{\omega^2}{\omega_1^2})(\omega_{TMDI}^2 - \omega^2 + i2\zeta_{TMDI}\omega_{TMDI}\omega) - \frac{\beta + \mu}{\omega_1^2}(\omega_{TMDI}^2 + i2\zeta_{TMDI}\omega_{TMDI}\omega)\omega^2}\]
Optimum design for harmonic excitation  

- Design problem: given an undamped primary system with specific dynamic characteristics \((m_1, k_1, c_1=0)\), determine the TMDI parameters \((k_{TMDI}, c_{TMDI})\) which minimise the response of the primary system, given \(m_{TMDI}\) and \(b\).

- Solution - ‘Equal points’ design method: there exist two fixed points where the FRF curves intersect (noted \(P_1\) and \(P_2\)), independent of \(c_{TMDI}(\zeta_{TMDI})\).

- Optimum response is obtained if and only if there exists two local maxima and both have the same amplitude.

- Mass ratio \(\mu=0.1\),

- Inertance ratio \(\beta=0.1\),

- Frequency ratio \(\nu_{TMDI}=0.5\).
Optimum design for harmonic excitation Den Hartog approach

- Minimum response \( \iff \) (if and only if) \( |G_1(\omega)| \) has two local maxima with equal amplitudes at the stationary points \( P_1 \) and \( P_2 \).
- Enforce:

1) \( |G_1(\omega)| \) is independent of \( \zeta_{TMDI} \):
\[
\lim_{\zeta_{TMDI} \to \infty} |G_1(\omega)|^2 = \lim_{\zeta_{TMDI} \to 0} |G_1(\omega)|^2
\]

2) Equal amplitude at points \( P_1 \) and \( P_2 \) for the limit \( \zeta_{TMDI} \to \infty \):
\[
\lim_{\zeta_{TMDI} \to \infty} |G_1(\omega_{p1})| = \lim_{\zeta_{TMDI} \to \infty} |G_1(\omega_{p2})|
\]

3) \( |G_1(\omega)| \) is maximized locally at points \( P_1 \) and \( P_2 \):
\[
\left. \frac{\partial |G_1(\omega)|}{\partial \nu} \right|_{\omega=\omega_{p1}} = \left. \frac{\partial |G_1(\omega)|}{\partial \nu} \right|_{\omega=\omega_{p2}} = 0
\]

\[ \Rightarrow \text{For } b=0 \ (\beta=0) \ \text{optimum TMD design is retrieved} \]

- Optimum frequency ratio \( \nu_{TMDI} \) :
\[
\nu_{TMDI} = \frac{1}{1+\beta+\mu} \sqrt{(1+\mu)(2-\mu) - \mu\beta} \quad 2(1+\mu)
\]

- Optimum TMDI damping ratio \( \zeta_{TMDI} \) :
\[
\zeta_{TMDI} = \sqrt{\frac{\beta^2 \mu + 6 \mu(1+\mu)^2 + \beta(1+\mu)(6+7\mu)}{8(1+\mu)(1+\beta+\mu)[2+\mu(1-\beta-\mu)]}}
\]

- Dynamic amplification factor:
\[
\max_\omega \{|G_1(\omega)|\} = \sqrt{\frac{(1+\mu)(\beta+2\mu+2)}{\beta+\mu}}
\]

Marian/Giaralis, SCHM, (2014, 2016)
Optimum design for harmonic excitation: vibration suppression

- All FRF curves attain two local maxima of equal height at the frequencies $\omega_{p1}$ and $\omega_{p2}$ whose location depend on the ratio $\beta$.

For the same $m_{TMDI}$ mass, the inclusion of the inerter (TMDI vs. TMD):
- reduces the peak response of the primary structure (at $\omega_1$, $\omega_{p1}$ and $\omega_{p2}$),
- increases robustness to “detuning effects” - Large $\beta$ values - wider range of frequencies peak response reduction compared to the classical TMD.
- Practically, the inerter furnishes all the positive effects of increasing the attached mass without the negative effect of the added weight.
Optimum design for harmonic excitation: weight reduction

- Peak response amplitude for optimally designed TMDI normalized by the peak response amplitude of the optimally designed classical TMD ($\beta = b = 0$).

- TMDI reduction saturates as the ratio $\beta$ increases

- Incorporation of the inerter to the classical TMD system is more effective for vibration suppression for smaller attached masses $m_{TMDI}$.

Example: - $|G_1(\omega)| = 2.9$ if:

1) TMD solution: mass ratio $\mu = 0.63$
2) TMDI solution - mass ratio $\mu = 0.34$ (for $\beta = 0.1$)
Optimum design for white noise excitation

- Closed form expressions for TMDI parameters to minimize the variance of the relative displacement of undamped primary structures

\[
\sigma_1^2 = \int_{-\infty}^{+\infty} \left| G_1(\omega) \right|^2 S(\omega) d\omega \\
S(\omega) = S_0 \quad \text{(ideal white noise)}
\]

Impose:

\[
\frac{\partial \sigma_1^2}{\partial \nu} = 0 \quad \text{and} \quad \frac{\partial \sigma_1^2}{\partial \zeta_{TMD}} = 0 \Rightarrow
\]

TMDI

\[
\nu_{TMDI} = \frac{1}{1 + \beta + \mu} \frac{\sqrt{[\beta(\mu - 1) + (2 - \mu)(1 + \mu)]}}{2\sqrt{(1 + \beta + \mu)[\beta(1 - \mu) + (2 - \mu)(1 + \mu)]}}
\]

\[
\zeta_{TMDI} = \frac{\sqrt{[\beta + \mu] \sqrt{\beta(3 - \mu) + (4 - \mu)(1 + \mu)}}}{2\sqrt{(1 + \beta + \mu)[\beta(1 - \mu) + (2 - \mu)(1 + \mu)]}}
\]

\[
\sigma_{1,\min}^2 = \pi S_0 \omega_1 (1 + \mu) \sqrt{\frac{(1 + \mu)[\beta(3 - \mu) + (4 - \mu)(1 + \mu)]}{(\beta + \mu)(1 + \beta + \mu)}}
\]

TMD

\[
\nu_{TMD} = \frac{\sqrt{(1 - \mu / 2)}}{1 + \mu}
\]

\[
\zeta_{TMD} = \frac{\sqrt{\mu(1 - \mu / 4)}}{\sqrt{4(1 + \mu)(1 - \mu / 2)}}
\]

\[
\sigma_{1,\min}^2 = \pi S_0 \omega_1 (1 + \mu) \sqrt{\frac{(1 + \mu)(4 - \mu)}{\mu}}
\]

Marian/Giaralis, PEM, (2014)

Warburton, EESD, (1982)
Optimum design for white noise excitation

- Additional $m_{TMDI}$ mass values (coefficient $\mu$) required for achieving the same level of performance in terms of displacement response variance for white noise base excitation for the proposed TMDI configuration ($\beta > 0$) and classical TMD ($\beta = 0$)

Marian/Giaralis, PEM, (2014)
Equations of motion: N-storey TMDI equipped shear frame building structure

\[ M \ddot{x} + C \dot{x} + K x = M \delta a_g \]

\[ \delta = \begin{bmatrix} 1 & 1 & 1 & \ldots & 1 \end{bmatrix}^T \]

\[ M = \begin{bmatrix} m_{TMDI} + b & 0 & -b & 0 & \ldots & 0 \\ 0 & m_1 & 0 & 0 & \ldots & \vdots \\ -b & 0 & m_2 + b & 0 & \ldots & \vdots \\ 0 & 0 & 0 & m_3 & \ldots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & \ldots & \ldots & \ldots & \ldots & 0 & m_n \end{bmatrix} \]

- Mass matrix

\[ C = \begin{bmatrix} c_{TMDI} & -c_{TMDI} & 0 & \ldots & 0 \\ -c_{TMDI} & c_1 + c_{TMDI} & -c_1 & 0 & \vdots \\ 0 & -c_1 & c_1 + c_2 & -c_2 & \vdots \\ 0 & 0 & -c_2 & \ddots & 0 \\ \vdots & \vdots & \vdots & \ddots & -c_{n-1} \\ 0 & \ldots & 0 & -c_{n-1} & c_{n-1} + c_n \end{bmatrix} \]

- Damping matrix

\[ K = \begin{bmatrix} k_{TMDI} & -k_{TMDI} & 0 & \ldots & 0 \\ -k_{TMDI} & k_1 + k_{TMDI} & -k_1 & 0 & \vdots \\ 0 & -k_1 & k_1 + k_2 & -k_2 & \vdots \\ 0 & 0 & -k_2 & \ddots & 0 \\ \vdots & \vdots & \vdots & \ddots & -k_{n-1} \\ 0 & \ldots & 0 & -k_{n-1} & k_{n-1} + k_n \end{bmatrix} \]

- Stiffness matrix

\[ \mathbf{x} = \begin{bmatrix} x_{TMDI}(t) & x_1(t) & x_2(t) & \ldots & x_n(t) \end{bmatrix}^T \]

- Displacements vector
Optimum design for evolutionary stochastic (seismic) excitation

Given:
- A primary structure with specific dynamic characteristics,
- An inerter with pre-specified inertance $b$,
- A pre-specified TMD mass:
  \[
  \mu = \frac{m_{TMDI}}{M_1} \quad M_1 = \{\phi_{1n}\}^T \left[ M_p \right] \{\phi_{1n}\}
  \]

Optimize for:
- TMD parameters: damping ratio ($\zeta_{TMDI}$) and frequency ratio:
  \[
  \zeta_{TMDI} = \frac{c_{TMDI}}{2(m_{TMDI} + b)\omega_{TMDI}} \quad \nu_{TMDI} = \frac{\omega_{TMDI}}{\omega_1}
  \]

Considering the objective function
\[
PI = J^{TMDI} / J^0, \quad J^{TMDI} = \int_{0}^{\infty} |G_1(\omega)|^2 \max_{t} \{S(\omega,t)\} \, d\omega
\]

$S(\omega,t)$ – evolutionary power spectral density (EPSD) function modelling the support excitation modelled by a stationary stochastic process

$|G_1(\omega)|^2$ - squared modulus of the frequency response function corresponding to peak floor displacement
Optimum design for evolutionary stochastic (seismic) excitation

Marian/Giaralis, PEM (2014)

- Equivalent mechanical model of the proposed frame structure. Mechanical admittance formulation.
- Mechanical admittances

\[ Q_1(s) = \frac{k_1}{s} + c_1, \quad Q_2(s) = \frac{k_2}{s} + c_2, \quad Q_3(s) = \frac{k_3}{s} + c_3, \quad Q_{TMD}(s) = \frac{k_{TMD}}{s} + c_{TMD}, \quad \text{and} \quad Q_{TTF}(s) = bs \]

- Equations of motion in Laplace form:

\[ [B][\ddot{x}] = -[M][\dot{\delta}]\ddot{A}, \]

- Admittance matrix \([B]\):

\[
[B] = \begin{bmatrix}
  m_1s^2 + [Q_1(s) + Q_{TMD}(s)]s & -Q_1(s)s & 0 & -Q_{TMD}(s)s \\
  -Q_1(s)s & m_2s^2 + [Q_1(s) + Q_2(s) + Q_{TTF}(s)]s & -Q_2(s)s & -Q_{TTF}(s)s \\
  0 & -Q_2(s)s & m_3s^2 + [Q_2(s) + Q_3(s)]s & 0 \\
  -Q_{TMD}(s)s & -Q_{TTF}(s)s & 0 & m_{TMD}s^2 + [Q_{TMD}(s) + Q_{TTF}(s)]s
\end{bmatrix}
\]

- Compute overall system transfer function:

\[ G_1(s) = \frac{\ddot{x}_1(s)}{\ddot{A}(s)}, \]
Considered input evolutionary stochastic seismic excitation

Modeling of the seismic excitation
(EPSD compatible with the Eurocode 8 elastic response spectrum)

\[ S(\omega, t) = C t \exp \left( \frac{-bt}{2} \right) \frac{1 + 4\zeta_g^2 \left( \frac{\omega}{\omega_g} \right)^2}{\left[ 1 - \left( \frac{\omega}{\omega_g} \right)^2 \right]^2 + 4\zeta_g^2 \left( \frac{\omega}{\omega_g} \right)^2} \frac{1}{\left[ 1 - \left( \frac{\omega}{\omega_f} \right)^2 \right]^2 + 4\zeta_f^2 \left( \frac{\omega}{\omega_f} \right)^2} \]

<table>
<thead>
<tr>
<th>C (cm/sec^{2.5})</th>
<th>b (1/sec)</th>
<th>\zeta_g (rad/sec)</th>
<th>\omega_g (rad/sec)</th>
<th>\zeta_f</th>
<th>\omega_f (rad/sec)</th>
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<tr>
<td>17.76</td>
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<td>0.78</td>
<td>10.73</td>
<td>0.90</td>
<td>2.33</td>
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</table>

Eurocode 8 compatible seismic design of TMDI equipped building structures

Range of different 3-storey building frames

<table>
<thead>
<tr>
<th>Structure</th>
<th>Story</th>
<th>Mass (kg)</th>
<th>Stiffness (N/m)</th>
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<tbody>
<tr>
<td><strong>REGULAR IN ELEVATION</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3DOF (I)</td>
<td>1 (top)</td>
<td>$30 \times 10^3$</td>
<td>$30 \times 10^6$</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>$30 \times 10^3$</td>
<td>$30 \times 10^6$</td>
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<tr>
<td></td>
<td>3</td>
<td>$30 \times 10^3$</td>
<td>$30 \times 10^6$</td>
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<tr>
<td><strong>WITH REDUCED TOP FLOOR STIFFNES</strong></td>
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<td></td>
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</tr>
<tr>
<td>3DOF (IIa)</td>
<td>1 (top)</td>
<td>$30 \times 10^3$</td>
<td>$25 \times 10^6$</td>
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<td>$30 \times 10^3$</td>
<td>$30 \times 10^6$</td>
</tr>
<tr>
<td>3DOF (IIb)</td>
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<td>$20 \times 10^6$</td>
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<td>2</td>
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<tr>
<td><strong>WITH INCREASED TOP FLOOR MASS</strong></td>
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<tr>
<td>3DOF (IIIa)</td>
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<td>$30 \times 10^3$</td>
<td>$30 \times 10^6$</td>
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<td>$30 \times 10^6$</td>
</tr>
<tr>
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<td>2</td>
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<table>
<thead>
<tr>
<th>Structure</th>
<th>Mode</th>
<th>Period (s)</th>
<th>Frequency (rad/s)</th>
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<tr>
<td>3DOF (IIIb)</td>
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</table>
Performance in terms of top floor displacement variance

Optimisation method:
- MATLAB® built-in “min-max” constraint optimization algorithm employing a sequential programming method

\[ 0.5 < \nu_{TMDI} < 1.10 \quad \text{and} \quad 0 < \zeta_{TMDI} < 1.00 \]

Eurocode 8 compatible seismic design of TMDI equipped building structures
Eurocode 8 compatible seismic design of TMDI equipped building structures

- Weight reduction

- Mass reduction: 3820 Kg for $b=0$ Kg (TMDI), 140000 Kg for $b=140000$ Kg (TMDI)
- Mass reduction: 5020 Kg for $b=0$ Kg (TMDI), 140000 Kg for $b=140000$ Kg (TMDI)
- Mass reduction: 6410 Kg for $b=0$ Kg (TMDI), 140000 Kg for $b=140000$ Kg (TMDI)
- Mass reduction: 7000 Kg for $b=0$ Kg (TMDI), 140000 Kg for $b=140000$ Kg (TMDI)

Performance Index - 3DOF (P.I.) vs. Additional $m_{TMDI}$ mass (Kg)
Eurocode 8 compatible seismic assessment of TMDI equipped building structures

TMDI assessment using Eurocode 8 compatible accelerograms

Giaralis/Spanos, SDEE (2009)
# Eurocode 8 compatible seismic assessment of TMDI equipped building structures

## Table 5. Maximum top floor displacements (cm)

<table>
<thead>
<tr>
<th>Ground acceleration no.</th>
<th>#1</th>
<th>#2</th>
<th>#3</th>
<th>#4</th>
<th>#5</th>
<th>#6</th>
<th>#7</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Primary structure</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TMD</td>
<td>8.21</td>
<td>10.68</td>
<td>8.28</td>
<td>9.87</td>
<td>9.91</td>
<td>7.60</td>
<td>7.85</td>
<td><strong>8.91</strong></td>
</tr>
<tr>
<td>(m=5400 Kg, b=0 Kg)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TMDI</td>
<td>4.86</td>
<td>4.79</td>
<td>5.51</td>
<td>6.38</td>
<td>5.01</td>
<td>4.97</td>
<td>5.65</td>
<td><strong>5.30</strong></td>
</tr>
<tr>
<td>(m=5400 Kg, b=80000 Kg)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TMDI</td>
<td>5.54</td>
<td>5.09</td>
<td>4.72</td>
<td>5.23</td>
<td>4.43</td>
<td>5.52</td>
<td>6.68</td>
<td><strong>5.31</strong></td>
</tr>
<tr>
<td>(m=5400 Kg, b=100000 Kg)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

## Table 6. Maximum top floor acceleration (g)

<table>
<thead>
<tr>
<th>Ground acceleration no.</th>
<th>#1</th>
<th>#2</th>
<th>#3</th>
<th>#4</th>
<th>#5</th>
<th>#6</th>
<th>#7</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Primary structure</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TMD</td>
<td>1.59</td>
<td>2.25</td>
<td>1.53</td>
<td>1.97</td>
<td>1.97</td>
<td>1.50</td>
<td>1.58</td>
<td><strong>1.77</strong></td>
</tr>
<tr>
<td>(m=5400 Kg, b=0 Kg)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TMDI</td>
<td>1.07</td>
<td>0.97</td>
<td>0.92</td>
<td>1.22</td>
<td>1.42</td>
<td>0.99</td>
<td>1.12</td>
<td><strong>1.10</strong></td>
</tr>
<tr>
<td>(m=5400 Kg, b=80000 Kg)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TMDI</td>
<td>0.57</td>
<td>0.44</td>
<td>0.44</td>
<td>0.40</td>
<td>0.34</td>
<td>0.45</td>
<td>0.55</td>
<td><strong>0.46</strong></td>
</tr>
<tr>
<td>(m=5400 Kg, b=100000 Kg)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TMDI</td>
<td>0.33</td>
<td>0.35</td>
<td>0.38</td>
<td>0.31</td>
<td>0.37</td>
<td>0.32</td>
<td>0.30</td>
<td><strong>0.34</strong></td>
</tr>
<tr>
<td>(m=5400 Kg, b=120000 Kg)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>0.30</strong></td>
</tr>
</tbody>
</table>

CITY UNIVERSITY LONDON
The TMDI suppresses the higher modes of vibration, not just the fundamental mode as the TMD does…

- Uncontrolled structure
- \( b=0 \) - TMD (Optimally designed)
- \( b=120000 \) - TMDI (Optimally designed)
Equations of motion: N-storey TMDI equipped shear frame building structure
Marian/Giaralis, PEM (2013, 2014)

\[ \mathbf{M} \ddot{\mathbf{x}} + \mathbf{C} \dot{\mathbf{x}} + \mathbf{K} \mathbf{x} = \mathbf{M} \delta a_g \]

\[ \delta = \{1 \ 1 \ 1 \ \cdots \ 1 \}^T \]

**Mass matrix** \( \mathbf{M} \)

**Damping matrix** \( \mathbf{C} \)

**Stiffness matrix** \( \mathbf{K} \)

- \( \mathbf{x} = \{ x_{TMD}(t) \ x_1(t) \ x_2(t) \ \cdots \ x_n(t) \}^T \) - Displacements vector

- \( m_{TMDI} + b \)
- \( b \)
- \( m_1 \)

- \( c_{TMDI} \)
- \( -c_{TMDI} \)
- \( c_1 + c_{TMDI} \)
- \( -c_1 \)

- \( k_{TMDI} \)
- \( -k_{TMDI} \)
- \( k_1 + k_{TMDI} \)
- \( -k_1 \)

TMDI for MDOF primary structures
Reliability-based optimum design framework for TMDI equipped seismically excited building structures

- **Alternative connectivity arrangements**
  \[
  \left( M_s + R_d m_d R_d^T + R_c bR_c^T \right) \ddot{x}_s + \left( m_d R_d + bR_c \right) \dot{y} \\
  + C_s \dot{x}_s + K_s x_s = - \left( M_s + R_d m_d R_d^T \right) R_s \ddot{x}_g
  \]
  and
  \[
  (m_d + b) \ddot{y} + \left( m_d R_d^T + bR_c^T \right) \ddot{x}_s + c_d \dot{y} + k_d y = -m_d R_d^T R_s \ddot{x}_g
  \]
  
  \( R_d \) : TMD location vector
  \( R_b \) : inerter location vector
  \( R_c = R_d - R_b \) : connectivity vector

- **Reliability-based design accounting for higher modes contribution**: Minimize the probability that any interstorey drift, or total floor acceleration, or the attached mass displacement (stroke) outcrosses a predefined threshold within the duration of stationary seismic excitation
  

- **Inertance b is treated as a free/design parameter**

- **Seismic input and structural uncertainty can be accounted for**
Reliability-based optimum design framework for TMDI equipped seismically excited building structures

\[ P_F(\varphi) = P(z(t) \notin D_s \text{ for some } t \in [0,T]) \]

**First-passage probability for vector output**

**Performance variables**
- interstorey drifts
- total floor accelerations
- attached mass stroke

**Design variables**
\[ \varphi = \{b, \omega_d, \zeta_d\} \]

**Augmented state-space matrix**
\[ \dot{x}(t) = A(\varphi)x(t) + E(\varphi)w(t) \]

**Output**
\[ z(t) = C(\varphi)x(t) \]

**First-passage probability for scalar output**
\[ B_i : |z_i(t)| = \beta_i \]

**Vector-output**
\[ D_s : \{z(t) \in \mathbb{R}^{n_z} : |z_i(t)| < \beta_i, \forall i = 1, \ldots, n_z\} \]

"Hyper-rectangular in the performance variables space"

Reliability-based optimum design framework for TMDI equipped seismically excited building structures

\[ P_F(\varphi) = P(\mathbf{z}(t) \notin D_s \text{ for some } t \in [0, T]) = 1 - \exp\left(-\nu_{z}^{+}(\varphi)T\right) \]

First-passage probability for vector output

Assumption: Stationary excitation and response

Reliability-based design

\[ \varphi^* = \arg \min_{\varphi \in \Phi} P_F(\varphi) \]

\[ \approx \arg \min_{\varphi \in \Phi} \nu_{z}^{+}(\varphi) \]

Unconditional outcrossing rate for vector output

\[ \nu_{z}^{+}(\varphi) = \lim_{\Delta t \to 0} \frac{E[\text{number of out-crossings in } [t, t + \Delta t] \mid \text{no out-crossings in } [0, t]]}{\Delta t} \]

Rice’s conditional outcrossing rate for scalar output

Correction factor for vector output, considering correlation of outcrossing’s at different parts of boundary

Temporal correlation factor correction for unconditional outcrossing rate

### Properties of the 10-storey TMDI equipped frame structure

<table>
<thead>
<tr>
<th>Stories</th>
<th>Storey Mass (ton)</th>
<th>Storey Stiffness $\tilde{k}_i$ (MN/m)</th>
<th>Mode</th>
<th>Period (s)</th>
<th>Participation factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-4</td>
<td>900</td>
<td>782.22</td>
<td>1\textsuperscript{st}</td>
<td>1.50</td>
<td>81.7%</td>
</tr>
<tr>
<td>5-7</td>
<td>900</td>
<td>626.10</td>
<td>2\textsuperscript{nd}</td>
<td>0.55</td>
<td>11.8%</td>
</tr>
<tr>
<td>8-10</td>
<td>900</td>
<td>469.57</td>
<td>3\textsuperscript{rd}</td>
<td>0.33</td>
<td>3.7%</td>
</tr>
</tbody>
</table>

Nominal modal damping: 3.5%

### Seismic Excitation: Clough-Penzien spectrum

$$S_g(\omega) = S_o \frac{\omega^4 + 4\zeta_g^2 \omega^2 \omega_g^2}{(\omega_g^2 - \omega^2)^2 + 4\zeta_g^2 \omega^2 \omega_g^2 (\omega_f^2 - \omega^2)^2 + 4\zeta_f^2 \omega_f^2 \omega^2} \omega^4$$

with mean values $\omega_g=3\pi$, $\omega_f=\pi/2$, $\zeta_g=0.4$, $\zeta_f=0.8$, $a_{RMS}=0.09g$

### Adopted thresholds

- Interstorey drifts: 3.3cm
- Floor acceleration: 0.5g
- Stroke: 1m

### Optimal out-crossing rate $\nu_z^+ (\phi^*) \cdot 100$ and optimal inerter ratio $\beta$ % (in parenthesis) for different TMDI topologies.

“TMD” denotes the cases for which $\beta=0$ is optimal.

<table>
<thead>
<tr>
<th>Topology</th>
<th>mass ratio $\mu_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.1%</td>
</tr>
<tr>
<td>$i_d$ $i_b$</td>
<td></td>
</tr>
<tr>
<td>10 9</td>
<td>1.507 (217.7)</td>
</tr>
<tr>
<td>10 8</td>
<td>0.348 (122.2)</td>
</tr>
<tr>
<td>9 10</td>
<td>1.526 (224.7)</td>
</tr>
<tr>
<td>9 8</td>
<td>0.626 (234.7)</td>
</tr>
<tr>
<td>9 7</td>
<td>0.071 (139.6)</td>
</tr>
<tr>
<td>8 9</td>
<td>0.634 (240.63)</td>
</tr>
<tr>
<td>8 10</td>
<td>0.355 (126.5)</td>
</tr>
<tr>
<td>8 7</td>
<td>0.276 (315.85)</td>
</tr>
<tr>
<td>8 6</td>
<td>0.024 (180.27)</td>
</tr>
<tr>
<td>7 8</td>
<td>0.278 (326.6)</td>
</tr>
<tr>
<td>7 9</td>
<td>0.072 (143.7)</td>
</tr>
<tr>
<td>7 6</td>
<td>0.135 (418.5)</td>
</tr>
</tbody>
</table>
Feasibility of inerters spanning multiple floors

Taipei 101 tower
Effectiveness of the TMDI to suppress higher modes

Normalized Transfer function for top floor acceleration

- uncontrolled
- TMD ($i_d=9$)
- TMDI ($i_d=9$, $i_b=8$)
- TMDI ($i_d=9$, $i_b=7$)
- TMDI ($i_d=8$, $i_b=10$)

$\omega_1$: fundamental structural frequency
Serviceability in wind-excited high-rise (tall) buildings

1. Discomfort level in terms of perception thresholds
2. Usually cross-wind vibration is critical for occupants comfort
3. The reference period for comfort evaluation is 1 year
4. $1^{st}$ natural frequency is dominant

Displacements
Loss of integrity of non-structural elements
Motion perception by building occupants

Scalar threshold:
- Office
- Apartment

Italian Guidelines

V(w(t;z), Vm(z1), Vm(z2), Vm(z3), V(t;z), u(t;z), v(t;z), X, Z, Y, θ, B1, B2, H)

Displacements
Accelerations

Loss of serviceability
Cross-wind forces due to vortex shedding

Vortex shedding phenomenon

Experimental results (wind tunnel testing)
Case study of a typical tall building subject to vortex shedding

Perimetric frame system  Central core  Complete FE model

Ciampoli/Petrini, PEM (2011)

Case study- Structure

- 74 floors
- Height $H=305m$
- Footprint $B_1=B_2=50m$
- Total mass = 92500 tons
For buildings having $H/B > 3$ the across-wind direction is dominant in terms of accelerations.

Due to vortex shedding effect

Compared to the along-wind direction, the across-wind acceleration $a_{LP}$ is significantly higher. This is evident in the peak response accelerations graph, where the across-wind acceleration is more pronounced. In the comfort evaluation graph, the across-wind direction shows a higher sensitivity to vibrations, indicating a poorer comfort level compared to the along-wind direction.
Considered vortex shedding input spectra

\[ \frac{nS_{f_y}(n)}{\sigma_j^2} = A \frac{H(C_1)n^2}{(1-n^2)^2 + C_1n^2} + C \]

Being

\[ H(C_1) = 0.179C_1 + 0.65\sqrt{C_1} \]

\[ C_1 = \left[0.47\left(\frac{D}{B}\right)^{2.8} - 0.52\left(\frac{D}{B}\right)^{1.1}\right] \]

\[ A = \left(H_{tot}/\sqrt{S}\right)\left[-0.118\left(\frac{D}{B}\right)^2\right] \]

\[ \sigma_j = \frac{1}{2}U(Z_j)^2B\overline{C_L} \]

And

\[ \overline{C_L} = 0.045\left(\frac{D}{B}\right)^3 - 0.335\left(\frac{D}{B}\right)^2 + 0.868\left(\frac{D}{B}\right) - 0.174 \]
Structural analysis in the frequency domain

Input/output relationship from random vibrations theory

\[ S_{QQ}(\omega) = B(\omega)^* S_{FFy}(\omega) B(\omega) \]

\[ S_{QQ}(\omega) = \omega^4 S_{QQ}(\omega) \]

Where

\[ S_{FFy}(\omega) = 2\pi S_{FFy}(n) \]

And

\[ B(\omega) = \left[ K - \omega^2 M + i\omega C \right]^{-1} \]

Response variances

\[ \sigma^2_{Q_j}(\omega) = \int_0^{\omega_{max}} S_{QQ_jQ_j}(\omega) d\omega \]

\[ \sigma^2_{\ddot{Q}_j}(\omega) = \int_0^{\omega_{max}} S_{\ddot{Q}_j\ddot{Q}_j}(\omega) d\omega \]

Peak responses

\[ Q_j^p = g \sqrt{\sigma^2_{Q_j}} \]

\[ \ddot{Q}_j^p = g \sqrt{\sigma^2_{\ddot{Q}_j}} \]

\[ g = \sqrt{2\ln(\eta T_{wind})} + \frac{0.577}{\sqrt{2\ln(\eta T_{wind})}} \]
Model reduction from detailed finite element model (FEM)

1) A diagonal mass matrix is assumed corresponding to a lumped mass 74 DOF system populated by the floor masses

2) Natural frequencies (eigenvalues) and mode shapes (eigenvectors) corresponding to the 74 lateral dominantly translational are obtained from the detailed FEM model

3) Derivation of a full stiffness matrix

\[ \begin{bmatrix} K_{(0)} - \omega_{(FEM)j}^2 M_{(0)} \end{bmatrix} \varphi_{(FEM)j} = 0 \]

with \( j=1,2,\ldots,74 \)

3) Derivation of a full damping matrix

\[ C_{(0)} = \left( \Phi^T \right)^{-1} C_{mod} \left( \Phi \right)^{-1} \]

\[ c_j = 2 \omega_j \xi_j \left( \varphi_j^T M_{(0)} \varphi_j \right) \]

<table>
<thead>
<tr>
<th>Mode j</th>
<th>Assumed damping ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-3</td>
<td>2%</td>
</tr>
<tr>
<td>4-6</td>
<td>4%</td>
</tr>
<tr>
<td>7-10</td>
<td>6%</td>
</tr>
<tr>
<td>11-20</td>
<td>9%</td>
</tr>
<tr>
<td>21-40</td>
<td>12%</td>
</tr>
<tr>
<td>41-60</td>
<td>15%</td>
</tr>
<tr>
<td>&gt;60</td>
<td>18%</td>
</tr>
</tbody>
</table>

Petrini/Giaralis (2016)
Case study of a typical tall building subject to vortex shedding

Independent parameters

| \[\beta=0:1\] | \[\mu=0.3\%; 0.5\%; 0.8\%; 1\%; 1.5\%\] |

Damping ratio

\[
\zeta_{TMDI} = \frac{(\beta + \mu)[1 + 0.75(\beta + \mu)]}{4(1 + \mu + \beta)[1 + 0.5(\beta + \mu)]}
\]

Frequency ratio:

\[
\nu = \sqrt{\frac{1 + 0.5(\mu + \beta)}{1 + \mu + \beta}}
\]

TMDI Connectivities

-1

-2

-3
Case study of a typical tall building subject to vortex shedding

Petrini/Giaralis (2016)

Top floor peak displacement

Top floor peak acceleration

TMDI connectivity
Case study of a typical tall building subject to vortex shedding

Petrini/Giaralis (2016)

Peak TMDI Stroke

![Graph of Peak TMDI Stroke]

Peak inerter forces

![Graph of Peak inerter forces]
### Sub-optimal inertance values and mass amplification factor

<table>
<thead>
<tr>
<th>Conn type</th>
<th>Response type</th>
<th>Parameter</th>
<th>Sub-optimal values of $\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\mu$=0.3%</td>
<td>$\mu$=0.5%</td>
</tr>
<tr>
<td>-1</td>
<td>Displ</td>
<td>$\beta$</td>
<td>0.1224</td>
</tr>
<tr>
<td>-2</td>
<td>Displ</td>
<td>$\beta$</td>
<td>0.2041</td>
</tr>
<tr>
<td>-3</td>
<td>Displ</td>
<td>$\beta$</td>
<td>0.5541</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Conn type</th>
<th>Response type</th>
<th>Parameter</th>
<th>Response amplification factors at sub-optimal values of $\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\mu$=0.3%</td>
<td>$\mu$=0.5%</td>
</tr>
<tr>
<td>-1</td>
<td>Displ</td>
<td>ampl factor</td>
<td>0.8523</td>
</tr>
<tr>
<td>-2</td>
<td>Displ</td>
<td>ampl factor</td>
<td>0.8339</td>
</tr>
<tr>
<td>-3</td>
<td>Displ</td>
<td>ampl factor</td>
<td>0.8108</td>
</tr>
</tbody>
</table>

The same response can be equivalently obtained by a classical TMD with $\mu=3.12\%$, and the same $\eta$, $\xi$ values.

**Mass/weight reduction by about 10 times**

Petrini/Giaralis (2016)
The TMDI exploits the mass amplification effect of the inerter to:

- improve the classical TMD performance for a fixed oscillating additional mass, or to
- “replace” part of the TMD vibrating mass by achieving an overall lighter passive control solution

This is an important consideration, especially for earthquake engineering applications and for tall buildings under wind excitations.

The TMDI controls more than one modes of vibration which may or may not be accounted for in the design.

This is an important consideration in suppressing floor accelerations in earthquake and wind excited buildings.
THANK YOU FOR YOUR ATTENTION

• Marian Laurentiu  City University London studentship (2012-2015)
  PhD thesis (viva 12/2016):
  *The tuned mass damper inerter for vibration suppression and energy harvesting in dynamically excited structural systems*

• EPSRC
  Big Pitch- Bright Ideas award (10/2014)
  *Multi-objective performance-based design of tall buildings using energy harvesting enabled tuned mass-damper-inerter (TMDI) devices*

  Grant Ref: EP/M017621/1

• Dr Francesco Petrini- EPSRC funded post-doctoral fellow
• Dr Jonathan Salvi- EPSRC funded post-doctoral fellow
• Assoc. Prof. Alexandros Taflanidis, University of Notre Dame, USA