The relationship of drug reimbursement with the price and the quality of pharmaceutical innovations

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Abstract: This paper studies the strategic interaction between pharmaceutical firms’ pricing decisions and government agencies’ reimbursement decisions which discriminate between patients by giving reimbursement rights to patients for whom the drug is most effective. We show that if the reimbursement decision precedes the pricing decision, the agency only reimburses some patients if the private and public health benefits from the new drug diverge. That is, when (i) there are large externalities of consuming the drug and (ii) the difference in costs between the new drug and the alternative treatment is large. Alternatively, if the firm can commit to a price in advance of the reimbursement decision, we identify a strategic effect which implies that by committing to a high price ex ante, the firm can force a listing outcome and make the agency more willing to reimburse than in the absence of commitment.

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Jel classification: I10, I18, L65.
1 Introduction

New drugs and medical devices are valuable goods that provide welcomed health benefits. However, they are also very expensive. According to the OECD, they are the major cost drivers of health care expenditures\(^1\). Given that in most countries the consumption of medicines is subsidized,\(^2\) the growth in the costs of new drugs has resulted in increasing public spending. In a situation where resources are limited and there is competition for public funds, the countries’ governments have had to find ways to rationalise the use and dissemination of these new products. For pharmaceutical goods, agencies have been set up to decide which drugs are "value for money". These agencies are constituted by committees of experts, who in consultation with the different parties (patients, providers and firms) decide which drugs are cost-effective. Some examples are: the National Institute for Clinical Excellence (NICE) in the UK, Pharmac in New Zealand, Fasi in Austria, the Pharmaceutical Benefits Advisory in Australia or the Commission of Transparence in France. The decisions these agencies take are based on clinical and economic evidence which is usually summarised in a cost effectiveness analysis. The analysis measures the health benefit associated with increasing access to a drug, places a monetary value on this benefit and compares it with the full cost of provision. In doing this the analysis identifies the group of patients for whom the drug will result in benefits that compensate the cost.\(^3\) The impact of a favourable cost-effectiveness test varies in each jurisdiction. It will usually imply that national guidelines are issued for public providers to encourage the use of the drug. In occasions, a positive result will directly imply that the drug is listed for reimbursement. For example, in the UK, since January 2002, the NHS is obliged to provide funding for NICE listed drugs after 3 months of the publication of the listing decision. Sometimes the effect of a positive result of a cost effectiveness on the listing of the drug will not be as immediate, but will feed in the considerations made in the decision making.

We model the strategic interaction between an innovator firm and a decision maker who chooses whether to list the drug for reimbursement and if so, which patients should be subsidized. The decision maker does this by comparing the

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\(^1\)For example, pharmaceutical expenditure has doubled in real terms in Sweden and Australia between 1990 and 2001, and increased by more than 70% in Canada, Finland, Ireland, and the US. See OECD, 2004.

\(^2\)Reimbursement policies vary from country to country. In the UK for example, patients must pay £6.40 per prescription. In Austria patients pay a fixed amount and a fraction of the price of the drug. In France patients pay 0%, 65% or 35% of the price of the drug, depending on the drug’s class.

\(^3\)For example, in March 2001, NICE considered Orlistat, a drug which fights obesity. The decision was that the drug should be prescribed to patients who had lost at least 2.5 kilos in weight by dieting and who either had a body mass index of 30 kg/m\(^2\) or a body mass index of 28 kg/m\(^2\) and the presence of significant co-morbidities. In the published report, one can find considerations about the 1998 direct costs and indirect costs to the NHS associated with obesity, a summary of the results of several clinical trials of Orlistat, and a cost effectiveness analysis which estimates the total annual drug costs of implementing the guidance, based on the current drug price.
excess health benefits and the excess costs of enlarging the patient group who is
granted reimbursement rights. The aim of this research is two-fold. Ultimately
we want to understand what are the effects of the listing decision on its provision
costs (public and private) and on the dissemination of the drug in the presence
of a strategic firm. Yet, to do this, we must study how the listing decision (and
its effects) depends on: the quality of the new drug with respect to existing
treatments, the existence of externalities associated with the consumption of the
drug and the extent to which they are considered by the agency, the possibility
that the firm commits to a price before the agency makes a listing decision and
the possibility that doctors prescribe the drug privately (aside the public
provider) generating an "unsubsidized" demand for the drug. We show that
if the reimbursement decision precedes the pricing decision, the agency only
reimburses some patients if the private and public health benefits from the new
drug diverge. That is, when (i) there are large externalities of consuming the
drug and (ii) the difference in costs between the new drug and the alternative
treatment is large. Alternatively, if the firm can commit to a price in advance
of the reimbursement decision, we identify a strategic effect which implies that
by committing to a high price ex ante, the firm can force a listing outcome and
make the agency more willing to reimburse than in the absence of commitment.

Whilst health economists and managers have paid a lot of attention to the
measurement of the benefits of drugs and the placement of their monetary value,
the analysis of the costs of provision has been highly neglected. With this work,
we point out that the cost of provision will crucially depend on how drugs’ prices
are set, as this will frame the ability of firms to react to the listing news. Whilst
agencies continue to base their analysis on historical prices and sales, they may
be miscalculating the costs of provision and making ill informed judgements.
Listing can have an impact on market variables and a prospective analysis is
needed. This implies understanding how firms react to and anticipate the news
that their drug is being listed for reimbursement and deriving the due effect on
market variables.

>From an academic point of view this research is also interesting as the
special feature of these agencies’ decision is that it is a decision by which “the
government” chooses to discriminate against (or for) a group of the population
which is not based on income levels but rather on how effective the product
to be subsidized is on the welfare of the group. This is the special feature of

\footnote{Patients also directly contribute to the financing of their consumption.}
\footnote{There are several sources of externalities: because of the nature of the disease- for example
infectious, because of knock on effects on the costs of providing health care, for example if
more consumption of the drug results in fewer hospitalizations and because of knock on effects
on families and social services budgets.}
\footnote{This will depend on the pricing regime. For example in the UK, the 1999-2004 Pharma-
caceutical Price Regulation Scheme which applies to all branded licensed NHS medicines implies
that pharmaceutical firms can initially choose the price at which they introduce a drug. How-
ever, after this, limited price changes must be approved by the Department of Health. This
approval is granted only if the company can proof that its return on capital is below 8.5%.
(For the new PPRS 2005- this is 8.4%) See publication in the Department of Health website.}
\footnote{In most countries, drugs can be prescribed by private health care providers and be bought
by patients who then benefit from no subsidy.}
our model as the impact of government agencies’ decisions to list medicines for patient reimbursement is an under-researched area which has focussed on reimbursement for low income patients rather than patients with high medical needs (see Scott Morton 1997). This difference is crucial and feeds in through the results.

Our paper is related to a small literature on drug formularies which analyzes health need based prioritization. Drug formularies are devices used by health care organizations to limit their expenditure on drugs. They basically list the drugs which consumers are to be reimbursed for, and, by exclusion, the drugs off the reimbursement list which may still be consumed, but are not subsidized. Olmstead at al (1999) analyze the optimal design of a drug formulary and Borrell (2003) studies the impact of the existence of drug formularies on drug prices. Our paper also analyzes the impact of an incentive based formulary on the prices of pharmaceutical innovations. However, unlike the above mentioned models we do consider the existence of strategic interactions between the agency and the innovator via prices.\(^8\)

A final word of caution is that this work has to be considered as a stepping stone of a larger research project. Health care provision is highly jurisdiction specific. Because of this any model of the health care industry suffers from the caveat that it will necessarily not apply everywhere. We have tried to overcome this problem by making the benchmark model as general as possible in so that it would still have interesting results and by pinning down what would be a worst case scenario for reimbursement. Our analysis is based on a situation where the firm is free to choose the initial price of the drug.\(^9\) where patients are differentiated by how effective the drug is on them and where the agency decides which patients to reimburse according to this.\(^10\) In the model, the agency’s objective depends on the private health benefits of patients, the non-private health benefits derived from the access to the drug (externality) and the public costs of provision. Consumers with no reimbursement rights are free to purchase the drug at full price (although they might not do so if it is expensive) and the firm can not price discriminate between consumers with reimbursement rights and others.

Section 2 presents the main features of the model. Sections 3 provides the solution to the Agency’s reimbursement problem and the firm’s pricing problem when the firm can not commit ex ante to a price. Section 4 extends the analysis to the situation where the firm can commit to a price before the Agency decides

\(^8\) Olmstead takes the agency’s costs due to the subsidy as exogenous, whereas Borrell works with a model with monopolistic competition, which is not suitable to analyze the case of innovators competing with lower quality off-patent products.

\(^9\) This is the government implements no regulation or constraint on the firm’s initial prices. This reflects the UK and US case. In other countries, there is some price bargaining between the government agency and the firm (for example, in France the Comité Économique du Medicament will negotiate the price and the volumes of reimbursed drugs, or in new Zealand, Pharmac will make the reimbursement decision contingent on some agreed price).

\(^10\) Indeed, from reading NICE’s published guidelines, one can see that often the selection of patients is based on a threshold of a diagnostic test, such as in the case of drugs for obesity, diabetes or Alzheimer’s disease, or on the description of certain symptoms.
on listing. Section 5 compares the two outcomes and finally, section 6 concludes.

2 The model

We model the strategic interaction between a pharmaceutical firm producing a new drug and a government agency that decides whose consumption should be subsidized.

The pharmaceutical firm launches a new drug of quality \( q, q > 1 \). This drug is patented and the only supplier is the firm. The marginal cost of production of the drug is constant and equal to \( c \). Patients differ in the improvements of health they derive from consuming the drug. In other words, the effectiveness of the drug depends on the patient type.\(^{11}\) By taking the new drug, patient type \( \theta \) benefits from an improvement in health of \( \theta \cdot q \). We assume that \( \theta \) is uniformly distributed in the interval \([\bar{\theta}, \bar{\theta}]\).

There is a second best alternative treatment which yields an improvement in health to type \( \theta \) of \( \theta \) only. This alternative is not listed for reimbursement and is supplied by a competitive fringe of firms at a given price of \( c \).\(^{12}\) We assume that \( \bar{\theta} > c \), that is, all patients will either purchase the innovation or the alternative treatment.

We consider a government agency with the power to decide which patient types can benefit from an exogenously determined subsidy on the price of the new drug.\(^{13}\) We define the amount of patients with reimbursement rights as the coverage level. An important aspect of our model is the assumption that the agency can treat patient types differently. This is the agency will select the coverage level according to the drug’s effectiveness on patients.

The agency chooses the coverage level so as to maximize its objective function. This objective function captures some of the observed features of the decision processes in a number of health agencies.\(^{14}\) In addition, the chosen objective function will serve as a benchmark in the sense that it will provide a worst case scenario for reimbursement. First, the agency’s objective function will not include the monetary costs borne by patients or the firm’s profits.\(^{15}\) This aims to reflect the absence of such considerations in the case of the UK Agency’s public reports, and also implies that our model does not favour a high level of coverage. In her decision making, the agency does not internalize

\(^{11}\)The same drug may result in differing health benefit for patients, depending on the severity and strand of the illness and the possibility of side effects. Patient heterogeneity was also considered in Olmstead and Zeckhauser (1999).

\(^{12}\)We can interpret this price as the cost of production of an alternative drug or as the cost of an alternative treatment for the patient. For example, an alternative to taking an obesity drug is dieting.

\(^{13}\)Subsidies are decided by general law and can not be changed for specific drugs.

\(^{14}\)Although we have looked in detail at the public reports of one such agency, the National Institute for Clinical Evidence in the UK, our intention is not to model the UK case in particular, but to abstract from it and make more general arguments which will inform this and other cases.

\(^{15}\)Note that this implies that the agency is not maximizing a welfare function.
the fact that as the coverage increases, profits increase and costs to patients decrease.

In taking her decision, the agency considers the public costs of reimbursement, the private health benefits from the consumption of the different available treatments—new and old (enjoyed by patients with reimbursement rights and by patients with no reimbursement rights) and an externality associated with the dissemination of the new drug. We model the externality as a per capita externality. We may interpret it as the public health benefits related to the consumption of the drug\textsuperscript{16} or the savings accrued by the health care system in terms of forgone on-going costs associated with the distribution of the drug.\textsuperscript{17}

Taking the stand of Olmstead and Zeckhauser (1999), who describe that “the goal in health care, at least implicitly, is to spend treatment dollars where they will produce significant benefits”, we assume that the agency decides to reimburse the patients for whom the drug is most effective, i.e. those characterized by a larger $\theta$. More specifically, the agency chooses a threshold $\theta_L$, such that all patients with $\theta > \theta_L$ benefit from the subsidy. We define the coverage level as $\theta - \theta_L$. The objective function of the agency is described below:\textsuperscript{18}

$$\int_{\theta_L}^{\theta} \theta \cdot q + \int_{\theta_L}^{\theta} \left( (P - S) + v \cdot (\theta - \theta_L) \right)$$

where $P$ is the full price of the drug, $S$ is the price paid by patients with reimbursement rights, and $v$ is the per capita externality.

We consider a general form for the consumer price: $S = \tau + \eta \cdot P$, with $\tau/(1 - \eta) \leq c$. Here, $\tau$ represents a flat rate and $\eta$ a proportional rate. Consequently, the cost per dose for the public funds is: $P - S = (1 - \eta)P - \tau$.

The pharmaceutical company chooses the drug’s price. The firm can freely choose the price but can not price discriminate between patients with and without reimbursement rights. Patients with no reimbursement rights can purchase the new drug at full price if they wish to.\textsuperscript{19}

In the following two sections, we analyze two alternative timings for the timing of events. First, we consider the case in which the coverage decision precedes the firm’s pricing decision. Second, we analyze the game where the firm is able to commit to a price before the coverage decision takes place (price

\textsuperscript{16}The existence of such externalities becomes evident in certain conditions like infectious diseases, serious mental illnesses and conditions involving long term incapacity, but, in general most medical conditions may in principle have an impact over labour productivity (see e.g. Francis (1997), Krieg (2002) and Laux (2000)).

\textsuperscript{17}Due to the reduction in hospitalization episodes, specialist needs and other costs.

\textsuperscript{18}Assuming that all patients with reimbursement rights purchase the drug and that patients' with no reimbursement rights do not purchase the drug.

\textsuperscript{19}All of these assumptions also reinforce a "worst case scenario for coverage". The fact that the firm is free to chose its price and that the firm is unable to price discriminate between patients with reimbursement and patients with no reimbursement rights both go in this direction. Similarly, the fact that patients with no reimbursement rights can purchase the drug privately as well as there is little need to subsidise the product if this is bought by consumers anyway.
commitment). We find the subgame perfect equilibrium of these two games by using backwards induction and we then compare the outcomes. All of the computations for the results can be found in a mathematical appendix.

3 Benchmark: The game with no price commitment

In this situation, in stage 1 the agency decides the coverage level $\theta - \theta_L$, in stage 2 the firm chooses its price and in stage 3 patients make their consumption decisions.

3.1 Stage 3: Patient’s consumption decisions

There are two groups of patients: those with reimbursement rights (if $\theta$ is such that $\theta > \theta_L$), and those without reimbursement rights (if $\theta$ is such that $\theta < \theta_L$). Patients with reimbursement rights buy the new drug if their utility $(\theta \cdot q - S)$ exceeds the utility they would obtain from the alternative treatment $(\theta - c)$.

The indifferent consumer is given by:

$$\theta_R = \frac{S - c}{\Delta q},$$

where $\Delta q = q - 1$. Similarly, for patients with no subsidy the indifferent consumer is:

$$\theta_F = \frac{P - c}{\Delta q} \cdot q.$$

Hence, the demand function for the new drug is:

$$D(P) = \begin{cases} \bar{\theta} - \theta_R, & \text{if } P > \frac{\Delta q \cdot \theta_L + c - \tau}{\eta} \theta_L \\ \bar{\theta} - \theta_L, & \text{if } \frac{\Delta q \cdot \theta_L + c - \tau}{\eta} \theta_L \geq P > \Delta q \cdot \theta_L + c \\ \bar{\theta} - \theta_F, & \text{if } P < \Delta q \cdot \theta_L + c. \end{cases}$$

Figure 1 depicts demand function for the new drug at a given threshold $\theta_L$.

Note that the demand curve has kinks and that its elasticity depends on whether consumers have access to the subsidy or not. As indicated in Figure 1, the demand function is more inelastic in the range where the price subsidy applies. It also has a range where it is completely inelastic, corresponding to prices such that $\theta_R < \theta_L < \theta_F$.

Definition 1 $\pi_R = (P - c)(\bar{\theta} - \theta_R)$ is the profit function when all patients have reimbursement rights (full coverage: $\theta_L = \bar{\theta}$) and $\theta^*_R = \frac{\Delta q \cdot \theta_L + c - \tau}{2 \Delta q}$ is the indifferent consumer that maximizes $\pi_R$. Similarly $\pi_F = (P - c)(\bar{\theta} - \theta_F)$ is the profit function when no patients have reimbursement rights ($\theta_L = \bar{\theta}$) and $\theta^*_F = \frac{\Delta q \cdot c - \tau}{2 \Delta q}$ is the indifferent consumer that maximizes $\pi_F$.

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20 Note that there is a divergence between consumer choice and the choice desired by the agency who would hope that patients internalized the externality in their decisions. This would be the case if the equation driving patients behavior was: $(\theta \cdot q + v - S) > (\theta - c)$. 

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Note that as long as the threshold $\theta_L$ is larger than $\theta^{R*}$, $\theta^{R*}$ will be the indifferent consumer driving demand for the new drug. Similarly, $\theta^{F*}$ will be the indifferent consumer provided it falls below the threshold $\theta_L$.

3.2 The firm’s choice of price

In this section, we characterize the price that maximizes the firm’s profits, $P^*$.\(^{21}\) The following proposition summarizes the main result:

**Proposition 2** There are three cases:

(i) **High coverage.** If $\theta_L < \theta^{R*}$, then $P^* = P_R = \frac{1}{2\eta}(\Delta q\bar{\mu} + L + \eta c - \tau)$, and $(\bar{\theta} - \theta^{R*})$ consumers purchase the drug. That is only some consumers with reimbursement rights purchase the drug.

(ii) **Intermediate coverage.** If $\theta^{R*} < \theta_L < \alpha$, then $P^* = P_L = \frac{\theta_L \Delta q\bar{\mu} + c - \tau}{\eta}$ and $\bar{\theta} - \theta_L$ consumers purchase the drug: All consumers with reimbursement rights purchase the drug.

(iii) **Low coverage.** If $\theta_L > \alpha$ then $P^* = P_F = \frac{1}{2}(\Delta q\bar{\mu} + L + c)$ and $(\bar{\theta} - \theta^{F*})$ consumers purchase the drug, even consumers with no reimbursement rights purchase the drug.\(^{22}\)

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\(^{21}\)In the appendix we find the local maxima in each of the demand regions (interior or corner solutions) and then we compare those maxima to obtain a global maximum.

\(^{22}\)The value of $\alpha$, ($\alpha > \theta^{F*}$), can be found in the Appendix. It is defined as the value of $\theta_L$ such that $\pi_F(\theta^{F*}) = \pi_R(\theta_L)$. 

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**Figure 2** provides a graphical representation of the result. The thicker function illustrates the maximum value of the firm’s profit as a function of the reimbursement level set by the agency, $\pi(\theta_L)$. The two other functions represent $\pi_R$ and $\pi_F$, as defined.

In order to understand proposition 2, it is worth acknowledging that the subsidy creates a wedge between the willingness to pay of consumers with reimbursement rights and the willingness to pay of consumers with no such rights. This implies that in order to serve consumers with no rights, the firm must reduce its price substantially (so as to make the indifferent consumer $\theta^{F*}$ fall below $\theta_L$). It will only pay to do so if the number of consumers with reimbursement rights is sufficiently low. The shape of the thicker curve in Figure 2 can now be explained.

If coverage is large $\theta_L < \theta^{R*}$, the firm can reach the highest level of profit $\pi^*_R$ as all the consumers who the firm serves have reimbursement rights. However, if coverage is smaller ($\theta^{R*} < \theta_L < \alpha$), it pays for the firm to adjust her prices (setting them high) so as to serve only those consumers who have reimbursement rights. Yet, there is a level of coverage ($\theta_L = \alpha$) below which it is not profitable for the firm to take notice of the small group of consumers with reimbursement rights. Figure 2 illustrates the maximum value of the firm’s profit as a function of the reimbursement level set by the agency, $\pi(\theta_L)$. The two other functions represent $\pi_R$ and $\pi_F$, as defined.

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\(^{21}\)In the appendix we find the local maxima in each of the demand regions (interior or corner solutions) and then we compare those maxima to obtain a global maximum.

\(^{22}\)The value of $\alpha$, ($\alpha > \theta^{F*}$), can be found in the Appendix. It is defined as the value of $\theta_L$ such that $\pi_F(\theta^{F*}) = \pi_R(\theta_L)$. 

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rights when setting prices. In this range, the best the firm can do is to lower prices so that consumers with no rights purchase the drug as well.

The impact of listing on the access to the new drug will depend on the level of coverage. If \( \theta_L \) is sufficiently high (\( \theta_L > \alpha \)), the firm will decide to serve \( \bar{\theta} - \theta_{FS} \) patients, where only a few (\( \bar{\theta} - \theta_L \)) will benefit from the subsidy. In this case, listing the drug will not result in a larger number of patients consuming it. Only \( \bar{\theta} - \theta_{FS} \) consumers would consume the drug, as if no subsidy existed.

For intermediate levels of coverage, \( \theta_{FS} < \theta_L < \alpha \), listing has perverse effects. Comparing this with the situation with no listing: the prices and the costs to the public funds are higher and only \( \theta_L - \theta_{FS} \) individuals purchase the drug, as opposed to \( \theta_L - \theta_{FS} \) (with \( \bar{\theta} - \theta_L < \bar{\theta} - \theta_{FS} \)).

Finally, with a high coverage level \( \theta_L \), listing results in an increase of the public costs but at the same time there is a larger consumption of the new drug. Which effect dominates will determine whether listing the drug is the best option for the agency or not.

### 3.3 The agency’s coverage decision

Given Proposition 2, it is clear that in choosing the subsidy’s coverage level, the agency indirectly selects the price regime. We identify the agency’s optimal choice. We first state the objective function for the agency, which, consistently with the analysis for the profit function, has a different form for the three different levels of coverage described in Proposition 2:\(^{23}\)

\[
OF(\theta_L) = \begin{cases} 
\int \pi \theta d\theta + \int \theta d\theta - \int \left( P_R - S_R \right) d\theta + v(\bar{\theta} - \theta_{RS}) & \text{if } \theta_L < \theta_{RS} \\
\int \pi \theta d\theta + \int \theta d\theta - \int \left( P_L - S_L \right) d\theta + v(\theta_L - \theta_{LS}) & \text{if } \theta_{RS} < \theta_L < \alpha \\
\int \pi \theta d\theta + \int \theta d\theta - \int \left( P_F - S_F \right) d\theta + v(\bar{\theta} - \theta_{FS}) & \text{if } \theta_L > \alpha 
\end{cases}
\]

where \( S_i = \tau + \eta P_i \).

Finally, the welfare of not listing the drug is:

\[
OF^{NL} = \int \pi \theta d\theta + \int \theta d\theta + v(\bar{\theta} - \theta_{FS}) = q\pi^2 - \theta_{FS}^2 - \Delta q(\theta_{FS})^2 + v(\bar{\theta} - \theta_{FS}).
\]

Note that the welfare of not listing coincides with the welfare achieved when there is no coverage \( \theta_L = \bar{\theta} \).

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\(^{23}\) An evaluation of the last expression can be found in the Appendix.
Proposition 3 If $\theta^* < \theta_L < \theta$, the agency does not reimburse any patients.

Note that since granting subsidies is costly, the agency only wishes to do so if there are added (private and public) health benefits. These added benefits only come across if by listing the drug for reimbursement, there is more access to the new drug. However, if $\theta_L > \theta^*$, listing the drug does not increase demand. As already explained, if $\theta^* < \theta_L < \alpha$, demand falls as a result of listing and if $\alpha < \theta_L < \theta$, demand is determined by $\theta^*$ and is unaffected by reimbursement. Since not reimbursing allows the agency to economise on costs, in these regions not listing is preferred by the agency to any coverage. In other words, from the point of view of the agency, there is no point in introducing a subsidy which will be available made available to people who would consume the drug even if that subsidy did not exist.\textsuperscript{24}

As a consequence, we only must check whether the agency would rather list the drug for reimbursement and set $\theta_L < \theta^*$ or not list the drug at all. In order to do this, we characterize the shape of the agency’s objective function. There are two forms for this function as depicted in Figures 3 and 4.\textsuperscript{25}

(INSERT FIGURES 3 AND 4 AROUND HERE)

The intuition behind these shapes is related to the impact of listing on the demand for the drug. Starting from no coverage ($\theta_L = \theta$), an increase in the level of coverage (a reduction in $\theta_L$) will initially reduce the agency’s payoff since, as explained, it will not raise the consumption of the new drug and will only increase the public costs (this explains the shape of $OF_3$). When $\theta_L = \alpha$ is reached, there is a downwards discontinuity in the $OF$ function. The discontinuity is due to the fact that at this point, the firm prefers to set higher prices and serve only the consumers with reimbursement rights rather than setting lower prices to serve $\theta - \theta^*$ consumers. The shape of the function for decreases in $\theta_L$ beyond $\alpha$ will depend on the strength of the subsidy. If the subsidy is low, further increases in coverage will actually increase the payoff to the agency (see $OF_2$ in Figure 3). For high subsidies a further increase in coverage may have an initial negative impact on the payoff to the agency (see $OF_2$ in Figure 4). In this range, the added health benefits of a larger coverage can not compensate for the increase in public costs. Finally, increases in coverage beyond $\theta - \theta^*$ will not have any impact on the agency’s payoff as for those levels demand for the new drug is fixed at $\theta - \theta^*$ (this explains the shape of $OF_1$).

In conclusion, the only global maximum candidates are either setting $\theta_L = \theta$ to achieve $OF^{NL}$ or setting any $\theta_L \in (\theta, \theta^*)$ to achieve $OF_1(\theta_L)$. We name these options not listing ($\theta_L = \theta$) and listing ($\theta_L \in (\theta, \theta^*)$). The following

\textsuperscript{24}If the agency cared about economic welfare (which would include private costs and the firm’s profit) there would be “more” listing. The reason is that when the agency does not list the drug, this results in larger costs for patients and/or smaller profits for firms. Given our specification of the agency’s objective function, these negative effects are not internalised by the agency’s decision who decides not to list the drug excessively from a welfare point of view.

\textsuperscript{25}See Appendix for a formal proof of the shape of these figures.
expression is the difference in payoffs between listing and not listing, which we define as the incentive to list:\footnote{We use this terminology but we wish to make the readers aware that this is not a marginal incentive. The decision of the agency at this stage is binary: either to list or not to list.}

\[
OF_1(\theta_L) - OF^{NL} = \frac{\Delta q}{2} \left( \left( \theta^{F*} \right)^2 - \left( \theta^{R*} \right)^2 \right) + \nu (\theta^{F*} - \theta^{R*}) - (P_R - S_R) \cdot (\overline{\theta} - \theta^{R*}).
\]

Since \( \theta^{F*} > \theta^{R*} \), the expression shows that the health benefits of listing exceed those of not listing, but as well that listing the drug is costly in terms of public funds. The balance of the health and the cost effects determines whether the agency decides to list.

>From here we deduce that \( \frac{\partial OF_1(\theta_L)}{\partial \theta^{R*}} - \frac{\partial OF^{NL}}{\partial \theta^{R*}} > 0 \). In other words, as the number of people who would buy the drug if it weren’t subsidized grows, the excess benefits of listing the drug for reimbursement are smaller and the incentives to list diminish. Indeed, the agency subsidizes the drug to supplement the lack of private demand. This is the case when \( \theta^{F*} \) is large as most patients would not buy the drug in the absence of a subsidy, and in this case, it might be that it is in the interest of the agency to list the drug for reimbursement and set \( \theta_L < \theta^{F*} \).\footnote{The derivative with respect to \( \theta^{R*} \) does not always have the same sign. The reason is that as \( \theta^{R*} \) grows the difference between the health benefits of listing the drug or not listing the drug are smaller, but it is unclear whether the costs of listing increase or not: on the one side, a larger \( \theta^{R*} \) is associated with a large price for the drug and a smaller public cost per dose, yet fewer patients are reimbursed and this might counterbalance the previous effect. The overall outcome is ambiguous.}

### 3.3.1 Comparative statics on the incentive to list, \( OF_1(\theta_L) - OF^{NL} \)

In this section, we check how \( OF_1(\theta_L) - OF^{NL} \), varies with \( v, c, \overline{\theta} \) and \( \Delta q \).

**Lemma 4** The incentive to list is more likely to be positive for large \( v \) and \( c \), and for small \( \overline{\theta} \).

A larger value of the externality results in a larger difference between the objective function in regime 1 and the objective function with no reimbursement. The effect is intuitive. Since more people access the drug with reimbursement, as the value of \( v \) grows, the larger is the effect on the difference between the objective functions.

As \( c - \overline{\theta} \) grows, the difference between access to the drug with and without listing increases. As a consequence, the difference between the values of the objective functions should increase. However, the effect of the cost difference on the difference in public costs is unclear. Given the sign of the derivative we can guarantee that even if public costs increase with listing, this adverse effect is overcome by the larger health benefits.
Lemma 5 There exists a $\eta^* \in (0, 1)$ such that if $\eta < \eta^*$, we obtain that the incentive to list is decreasing in $\Delta q$ and $\theta$. If $\eta > \eta^*$ then the incentive to list is increasing in $\theta$.

An increase in $\Delta q$ has a number of effects on the decision to list which we detail: On the health side, quality will have a positive direct impact on the incentives to list as for a given number of excess patients treated the benefits will be larger. Yet, the number of patients treated is not fixed, and there is an indirect negative effect through the changes in regime demands that a higher quality results upon. With higher quality more consumers purchase the drug with no listing, and the excess access of listing is smaller. Moreover changes in $\Delta q$ will also affect the public costs of listing. The increase in $\Delta q$ will have a positive impact of the unit price paid by the agency: 

$$\frac{\partial (P_R - S_R)}{\partial \Delta q} > 0$$

and in this way a positive impact on the listing costs (fewer incentives to list). However, depending on the sign of $\frac{\partial \theta R^*}{\partial \Delta q}$ this effect might be partially compensated by a reduction in the number of patients who have reimbursement rights. This is when $\frac{\partial \theta R^*}{\partial \Delta q} < 0$. The overall cost effect can therefore be ambiguous. However, in the case where subsidies are large, the overall effect is that a higher quality reduces the incentives to list.

The final lemma in this section reinforces the idea that mainly it is the wedge between private willingness to pay and public willingness to pay (determined by the externality) that creates a need to list the drug for reimbursement.

Lemma 6 If $0 < \eta < 1$, $v = 0$, $c = 0$ and $\tau = 0$, and $c < \frac{2\Delta q\theta}{(\eta + \sqrt{\eta^2 + 2\eta})}$ we obtain that $OF_1(\theta_L) - OF_{NL} < 0$. This is the agency opts for not listing.\textsuperscript{29}

4 The game with price commitment

In this section, we analyze the outcome of the coverage decision in the case where the firm can commit to a price in advance of the agency’s decision. In this game, in stage 1, the firm chooses the price, in stage 2, the agency chooses the coverage $\theta - \theta_L$, and in stage 3 consumers make purchasing decisions. For simplicity, we focus on the case where $v = 0$, $c = 0$ and $\tau = 0$, which allows a direct comparison with Lemma 6.\textsuperscript{30}

\textsuperscript{28}Whether there are more consumers purchasing the drug with listing depends on the sign of $\frac{\partial \theta R^*}{\partial \Delta q}$. However, given that $|\frac{\partial \theta R^*}{\partial \Delta q}| > |\frac{\partial \theta R^*}{\partial \Delta q}|$ even in the case where $\frac{\partial \theta R^*}{\partial \Delta q} \leq 0$, we find that listing the drug results in a smaller access with higher quality. The reason is that consumers purchasing decisions are more reflective of quality when consumers pay the full price.

\textsuperscript{29}Given $c < \frac{2\Delta q\theta}{(\eta + \sqrt{\eta^2 + 2\eta})}$, $\Delta q\theta > 2c$ is a sufficient condition for no listing for all $\eta$, with $\eta < 1$.

\textsuperscript{30}We have also showed in an extended version of the paper, that the results of this section are qualitatively very similar to the results that one would obtain if $\tau > 0$ and $\eta = 0$. However, note that in the absence of price commitment if $\eta = 0$ the profit maximization problem of the firm is unbounded as demand is fixed.
Note that, \( \theta_R = \frac{\eta P}{\Delta q} \) and \( \theta_F = \frac{P}{\Delta q} \). However, here the price \( P \) is an ex ante commitment and therefore, ex post it does not depend on whether the drug is reimbursed or not. The assumption that the firm can commit to a price generates a substantial difference in the outcome of the game. It turns out that, if the firm can commit to a price before the agency decides on coverage level, it can actually induce listing by choosing a sufficiently high price.\(^{31}\)

### 4.1 The agency’s coverage decision

For any given \( P \), there are three possible choices of \( \theta_L \):

\[
OF = \begin{cases} 
\int \theta q d\theta + \theta_R \int \theta d\theta - (1 - \eta) P(\bar{\theta} - \theta_R) & \text{if } \theta^F > \theta_R > \theta_L \\
\int \theta q d\theta + \theta_L \int \theta d\theta - (1 - \eta) P(\bar{\theta} - \theta_L) & \text{if } \theta^F > \theta_L > \theta_R \\
\int \theta q d\theta + \theta^F \int \theta d\theta - (1 - \eta) P(\bar{\theta} - \theta_L) & \text{if } \theta_L > \theta^F > \theta_R 
\end{cases}
\]

We first find the choice of \( \theta_L \) that maximizes the objective function of the agency, \( \theta^*_L \). This entails finding the local maxima for each of the three regions and comparing them. Figures 5 and 6 show the shape of the objective function and indicate the candidates for global maximum.

(INSERT FIGURES 5 AND 6 AROUND HERE)

The intuition for these shapes is the same as in section 3. The change in the timing of the game only affects the shape of \( OF_2 \). For a low subsidy (\( \eta > 1/2 \)), reductions in \( \theta_L \) will have a positive effect on the objective function of the agency. In this range (see Figure 5) listing increases the consumption of the new drug by \( (\theta^F - \theta_L) \) individuals. This positive impact is only partially offset by the negative effect of the increase in the public as the subsidy is low. As a result of a larger coverage there is an increase in the \( OF \). Contrarily, in the case of high subsidies (\( \eta < 1/2 \)), the cost effect dominates the access effect if \( \theta_L \) is low enough: when \( \theta_L > \theta^* \), \( OF_2 \) is decreasing in \( \theta_L \) (see Figure 6).

Hence, the global optimal coverage level, \( \bar{\theta} - \theta^*_L \), depends on the level of the subsidy. With a low subsidy (\( \eta > 1/2 \)) there might be no listing (\( \theta^*_L = \bar{\theta} \)) or listing with \( \bar{\theta} - \theta^*_L \) patients who purchase the drug (if \( \theta^*_L < \theta^R \)). With a high subsidy (\( \eta < 1/2 \)) there can be no listing (\( \theta^*_L = \bar{\theta} \)) or listing where \( \bar{\theta} - \theta^*_L \) patients purchase the drug with \( \theta^*_L > \theta^R \).

**Proposition 7** If \( \eta > 1/2 \) and \( P > \frac{2\Delta q}{1+\eta} \) or \( \eta < 1/2 \) and \( P > P_{LIM} = \frac{2(1-\eta)\Delta q}{2+\eta^2-2\eta} \), the agency lists the drug for reimbursement. Otherwise she does not.

\(^{31}\)In the UK, such commitment can be achieved because of the way in which pharmaceutical "prices" are regulated. According to the Price Regulation Scheme initially the firm is free to choose a price for the drug, but subsequent changes (especially increases) need to be approved by the Scheme. Very few changes have been approved.
By setting a sufficiently high price the drug company can guarantee a listing outcome. This might seem counterintuitive because as the price rises the unit cost of listing rises as well. However, it is also true that as price rises the difference between the excess health benefits of listing grow and do so at a quicker rate than the costs of listing. By committing to a high price the firm is actually "threatening" the agency with a very small level of access to the drug in the case where there is no reimbursement and all the demand is private. This makes the agency more willing to list, despite the larger cost.

4.2 The firm’s choice of price

In this setting the firm chooses the price taking into consideration the agency’s response. Clearly, low prices result in no coverage, and high prices result in some coverage. What matters for the choice of the optimal price $P^*$, is the comparison of the profits of each situation. Having some coverage will generally benefit the firm, unless the price rise needed to achieve this solution is so large that the resulting profit is smaller than the profit of no listing.

4.2.1 Low subsidies ($\eta > 1/2$)

In this case by forcing listing, the firm positions herself in $\pi_R$ as opposed to $\pi_F$ in Figure 2. This does not imply that profits are larger with listing as this really depends on how large the price threshold for the listing regime is.

**Proposition 8** If $\eta > 1/2$, the firm sets $P^* = \frac{\Delta q_\theta P + \eta c}{2\eta}$, which yields profits: $\frac{1}{15\Delta q} (\Delta q_\theta - \eta c)^2$, the drug is listed for reimbursement, and the demand for the new drug is $(\delta - \theta R^*)$. All consumers who purchase it are reimbursed.

If the subsidy is small, the firm can set the interior price that maximizes the profit function with listing, as this price is sufficiently high so that the agency lists the drug. Recall that lemma 6 tells us that with no price commitment and for $c < \frac{2\Delta q_\theta}{(\eta + \sqrt{\eta^2 + 2\eta})}$ the drug would not be listed for reimbursement. The question is why it is that with price commitment the agency decides to list. The reason is simple: by committing to a high price the firm is threatening the agency with limited access to the drug in the case that the drug is not listed. On the face of such event the agency decides to list.

4.2.2 High subsidies ($\eta < 1/2$)

With high subsidies the result is not as clear cut as Proposition 8. There are circumstances where it is in the firm’s interest to induce listing, and circumstances where this is not true.

With high subsidies if the drug is listed for reimbursement, the firm’s profits are: $(P-c) \left( \delta - \frac{(1-\eta)P}{\Delta q} \right)$. These profits achieve a maximum value of $\frac{1-\eta}{4\Delta q} \left( \frac{\Delta q_\theta}{1-\eta} - c \right)^2$.

\[\text{Note that } P^* = P_R \text{ for } \xi = 0 \text{ and } \tau = 0.\]
at $P^+ = \frac{c}{\eta} + \frac{\Delta q}{2(1-\eta)}$ and $\theta^+ = \frac{\Delta q\theta + (1-\eta)c}{2\Delta q}$. This profit value exceeds the maximum profit value when no patient holds reimbursement rights. Therefore, if $P^+$ is high enough to induce listing, the firm will set this price and induce listing. However, there will be situations in which $P^+$ is too small to induce listing and where the firm will need to generate an upwards distortion from the optimal price $P^+$ in order to induce listing. In some of these cases, it will be better for the firm not to induce listing. The following proposition presents the conditions under which the firm will induce listing.

**Proposition 9** For $\eta < 1/2$

9.1. If $c > c^* = \frac{\Delta q\theta^2(2+3\eta^2-6\eta)}{(1-\eta)(2+\eta^2-2\eta)}$, $P^* = P^+$. The drug is listed for reimbursement and $(\bar{\theta} - \theta^+)$ patients purchase it and are subsidized.

9.2. If $0 < c < c^*$, $P^+$ does not induce listing. In this case, the firm might choose $P_{LIM} = \frac{2(1-\eta)\Delta q\theta}{1-\eta}$ (the smallest price that induces listing, with an indifferent consumer $\theta_{LIM}$) or $P = \frac{1}{2}(\Delta q\theta + c)$ and induce no listing. The firm sets $P^* = P_{LIM}$ if:

$$\theta_{LIM} < z \text{ or } \theta_{LIM} - \theta^+ < \frac{\sqrt{\left(\Delta q\theta^2 - c^2(1-\eta)\right)}}{2\Delta q}$$

where $z$ is defined as the threshold value of $\theta$ that induces listing, this is $z \in (\theta^+, \bar{\theta})$ such that $\pi^L(z) = \pi^{NL^*} = \frac{1}{4\Delta q}(\Delta q\theta - c)^2$.

An illustration of the proposition above is provided in Figure 7.

**Corollary 10** A numerical simulation indicates that for $c = 0$, if $\eta$ is smaller than approximately $0.42$, the firm will choose $P = \frac{1}{2}(\Delta q\theta + c)$ and not induce listing, otherwise the firm will choose $P_{LIM}$ and induce listing.

## 5 Comparison of commitment and non commitment outcomes

In this section we compare the outcome with price commitment with the outcome described in lemma 6 (non price commitment). The following table summarises the comparison for all cases\textsuperscript{34,35}

\textsuperscript{33}Note that $c^* > 0$ only if $0 \leq \eta \leq 0.42$. Hence if $0.42 \leq \eta$ we have that $c^* < 0$ and only case 9.1 is relevant.

\textsuperscript{34}In the appendix we show that $\frac{2\Delta q\theta}{(\eta + \sqrt{\eta^2 + 2\eta})} > c^*$.

\textsuperscript{35}In this table we report the outcome where $c^* > 0$ (which can happen only if $0 \leq \eta \leq 0.42$). If $0.42 \leq \eta$, cases e and f in the table are not relevant.

(iinsert Table)
The table confirms the intuition that the firm’s price commitment results in more listing. This is due to the effect of the ex ante pushing of the prices which results in a listing outcome. In most circumstances (cases b, d and e) this benefits the firm who will commit to such high prices. However, if the subsidy is high and the cost is large (case c), the commitment of the firm results in a reduction of profits. The reason for this is that in this case in the absence of price commitment the drug would be listed anyway and there would be no rationing of the patients who can access it with a subsidy. Instead, with commitment as the price is higher, the agency rations the number of patients who have reimbursement rights and therefore the profits of the firm are smaller. In this case the firm will not commit to a price.

The comparison of the objective functions for the agency under commitment and no commitment tells the other side of the story. If the subsidy is small or if it is high but the costs of production are small, the absence of commitment favours the agency (these are cases b, e and partly d). If instead the subsidy is large and the production costs are large the commitment favours the agency.

6 Conclusions

This paper identifies the plausible effects of the strategic interaction between government agencies making decisions to subsidize patient consumption of drugs based on how effective these drugs are on different patient groups and firms making decisions about drug prices. We focus the analysis on the costs of drug provision, a relatively under-researched area, as most of the literature has considered the measurement of health benefits and its monetary value. Our remit is to make two simple points about this cost:

1. The cost of provision can not be based on historical prices and sales as the reimbursement decision may have an impact on market prices and quantities. A prospective analysis is needed.

2. Because of the former, it is crucial to understand how drug prices are set, and how firms react to and anticipate "reimbursement news".

The paper deals with these two points in a specific setting, which is an initial stepping stone of a larger research project where other cases should be analyzed. The analysis is based on a situation where patients can purchase the drug at full price if they have no reimbursement rights, where the pharmaceutical firm is free to choose prices but can not price discriminate between subsidized and unsubsidized consumers, and where the agency chooses a level of coverage (or group of consumers to subsidize) taking into consideration the excess health benefits of doing so (including private benefits and externalities) and balancing those against the excess public costs. In the model, the agency chooses an effectiveness threshold, and patients who fall in this range are subsidized.

In our benchmark scenario, the agency takes the listing decision first and then the firm chooses the price. Here, the main reason for listing the drug for reimbursement is to expand the benefits of the drug to consumers who would not purchase it privately, despite the public cost. Given the agency’s aim of
reducing public costs, subsidizing a few needy consumers makes no sense if those consumers would have purchased the drug at full price. Hence, the agency’s decision will be either to subsidize no patients at all, or to subsidize some, but in this case it will give reimbursement rights as well to patients who would not have purchased the drug at full price. It pays to do the latter whenever the private and public benefits from drug access really diverge—this is when consumers purchase too little privately. This might happen for several reasons: (i) patients are not willing to pay for the drug, but there are large externalities of them consuming the drug and (ii) the difference in costs between the new drug and the alternative treatment is large. Indeed, if there is the difference between the costs is small and there is no externality, the agency reimburses no consumers. It is also interesting to note that if the subsidy is high, the more effective the new drug is (higher quality), the fewer are the incentives to list it. The reason for this is that as the quality increases, the patients are more willing to purchase the drug privately.

In an alternative scenario, we study what the outcome would be if the firm could commit to a price before the agency decided on the listing decision. We prove that in this situation, for most parameter configurations, the firm decides to increase prices as this is a means to induce listing. The mechanism is the following: by committing to a high price ex ante, the firm is “promising” small access to the drug in the if there is no reimbursement. If the promise is credible, the agency is more willing to reimburse. We do not identify in the paper what makes the promise credible, but we want to note that regulations which imply some stickiness in prices will make those commitments possible. This is the case with the UK regulation for drug prices, the Pharmaceutical Price Regulation Scheme, which allows initial free pricing for a drug but then makes prices very difficult to change.

One must note that if private demand did not exist (for example because doctors were forbidden from recommending unlisted treatments) or because the cost of the drug would be so high that no consumer could purchase it at full price (which is part of our model), then the strategic effect of price increases with commitment would disappear. Increasing the price would not constitute a threat to diminish private demand and would only result in a larger public unit cost. In this situation, if in the absence of commitment, the agency would decide not to reimburse, then the firm would only "force" a listing outcome by reducing the price relative to the optimal price with reimbursement. This is the result would be the opposite to the one obtained. Our model then becomes more relevant in a world of mixed public/private provision of health care.

36 This is, in the end, the choice of an optimal coverage level, boils down to a binary decision: either the agency does not list, or it lists and gives reimbursement rights to a "large" amount of patients.
References


8 Appendix

Section 3.1: Full derivation of the firm’s demand function.

Since $\theta^F > \theta^R$, there are 3 regimes:

**Regime 1**, $(\theta_L < \theta^R)$: The demand of patients with reimbursement rights is $D^R = \overline{\theta} - \theta^R$. There is no demand of patients with no reimbursement rights.

**Regime 2**, $(\theta^R < \theta_L < \theta^F)$: The demand of patients with reimbursement rights is $D^R = \overline{\theta} - \theta_L$. There is no demand of patients with no reimbursement rights.

**Regime 3**, $(\theta^F < \theta_L)$: The demand of patients with no reimbursement rights is $D^R = \theta_L - \theta^F$. All patients with reimbursement rights purchase the drug: $D^R = \overline{\theta} - \theta_L$.

Finally the demand function is:

$$D = \begin{cases} \min \{\overline{\theta} - \theta^R, \overline{\theta} - \theta_L\} & \text{if } \theta^F > \theta_L \\ (\overline{\theta} - \theta_L) + (\theta_L - \theta^F) = \overline{\theta} - \theta^F & \text{if } \theta^F < \theta_L \end{cases}$$

**Proposition 2.**

We start by finding optimal decisions of the firm for each coverage regime:

**Regime 1**, $(\theta_L < \theta^R)$: The profit maximization problem is:

$$\max_{\{\theta^R\}} \pi_R(\theta^R) = \frac{1}{\eta} \left(\Delta \theta^R + \xi - \tau - \eta c\right) \left(\overline{\theta} - \theta^R\right)$$

such that $\theta^R > \theta_L$. The interior solution is $\theta^R^* = \frac{\Delta \theta + \xi + \eta \cdot c - \tau}{2\Delta \eta}$ and $P_R = \frac{1}{2\eta} \left(\Delta \theta + \xi + \eta \cdot c - \tau\right)$. The value of profits at this solution is: $\pi^*_R = \frac{1}{\eta} \frac{\Delta \theta + \xi + \eta \cdot c - \tau}{2\Delta \eta}^2$. Note that $\theta^R^* > \theta_L$ implies that $\Delta \theta + \xi + \eta \cdot c - \tau > \theta_L$ (condition A1).

**Regime 2**, $(\theta^R < \theta_L < \theta^F)$: The profit maximization problem is:

$$\max_{\{P\}} \pi_L(\theta_L) = (P - c) \left(\overline{\theta} - \theta_L\right)$$

Since $\pi_L(\theta_L)$ is increasing in $P$, the optimal price is the largest price that guarantees that consumer $\theta_L$ purchases the drug. Hence: $P_L = \frac{\theta_L \Delta \theta + \xi + \eta \cdot c - \tau}{2\Delta \eta}$. The value of profits at this solution is $\pi^*_L = \frac{\theta_L \Delta \theta + \xi + \eta \cdot c - \tau}{2\Delta \eta} \cdot \left(\overline{\theta} - \theta_L\right)$.

**Regime 3**, $(\theta^F < \theta_L)$: The profit maximization problem is:

$$\max_{\{\theta^F\}} \pi_F(\theta^F) = \left(\Delta \theta^F + \xi - c\right) \left(\overline{\theta} - \theta^F\right)$$

such that $\theta^F \leq \theta_L$. The solution is $P_F = \frac{1}{2} \left(\Delta \theta + \xi + c\right)$ and $\theta^F^* = \frac{\Delta \theta + \xi + c}{2\Delta \eta}$. The profits evaluated at this solution are: $\pi^*_F = \frac{1}{2\Delta \eta} \left(\Delta \theta + \xi + c\right)^2$. Note that $\theta^F^* \leq \theta_L$ is satisfied if $\frac{\Delta c}{2\Delta \eta} < \theta_L$ (condition A3).

**Boundary Conditions**
An interior solution in Regime 1 (respectively, Regime 3) requires A1 (A3) to hold. If A1 (A3) is not satisfied, the corner solution for the regime is a price such that \( \theta^R = \theta_L \) (respectively, \( \theta^F = \theta_L \)). This corner solution yields profits of \( \pi_R(\theta_L) = \frac{1}{\eta} (\Delta q \theta_L + \xi - \tau - \eta c) (\bar{b} - \theta_L) \), \( \pi_F(\theta_L) = (\Delta q \theta_L + \xi - c) (\bar{b} - \theta_L) \).

Note that, \( \theta^{R*} < \theta^{F*} \) since \( \tau/(1-\eta) < c < \Delta q \bar{b} \). Hence, A1 and A3 are incompatible. If one holds, the other does not. Note also that if, \( \theta^{F*} < \theta_L < \theta^{R*} \) neither \( \theta^{F*} \) nor \( \theta^{R*} \) are valid solutions. As a consequence we can establish the candidate solutions for each of the following cases:

**Case 1.** If \( \theta_L > \theta^R* \) we must compare: \( \pi_R(\theta_L), \pi^*_L \) and \( \pi^*_F \).

**Case 2.** If \( \theta^{F*} < \theta_L < \theta^{R*} \) we must compare: \( \pi_R(\theta_L), \pi^*_L \) and \( \pi_F(\theta_L) \).

**Case 3.** If \( \theta_L < \theta^{F*} \), we must compare: \( \pi_R, \pi^*_L \) and \( \pi_F(\theta_L) \).

Since \( \pi_R(\theta_L) = \pi^*_L > \pi_F(\theta_L) \), in Case 2 the global solution is \( \pi^*_L \). For the other cases, the comparison simplifies to: Case 3. \( \{\pi^*_R, \pi^*_L\} \) and Case 1. \( \{\pi^*_L, \pi^*_F\} \).

In Case 3 the global solution is \( \pi^*_R \) since \( \pi^*_R > \pi_R(\theta_L) = \pi^*_L \).

In Case 1 we obtain that there exists an \( \alpha \), with \( \alpha > \theta^{R*} \) such that if \( \theta_L > \alpha \) then the global solution is \( \pi^*_F \). Otherwise, the global solution is \( \pi^*_L \). The value of \( \alpha \) is:

\[
\alpha = \theta^{R*} + \sqrt{\left(\Delta q \bar{b} + \eta c + \tau - \xi\right)^2 - \left(\Delta q \bar{b} + \eta c + \tau - \xi\right)^2 - 4\Delta q \bar{b}(\tau + \eta c - \xi)}. \]

The proof of this last result is in three steps:

**Step 1.** If \( \theta_L = \bar{b} \), then \( \pi^*_L = 0 \) and \( \pi^*_F - \pi^*_L > 0 \).

**Step 2.** \( \pi^*_F - \pi^*_L = \frac{1}{\Delta q} \left(\Delta q \bar{b} + \eta c - \xi\right)^2 - \frac{1}{\eta} (\bar{b} - \theta_L) (\Delta q \theta_L + \xi - \tau - \eta c) \). Then:

\[
\frac{\partial(\pi^*_F - \pi^*_L)}{\partial \theta_L} = -\frac{1}{\eta} \cdot (-2\Delta q \cdot \theta_L + \Delta q \bar{b} + \eta c + \tau - \xi) = \frac{2\Delta q}{\eta} \left(\theta_L - \theta^{R*}\right). \]

**Step 3.** Finally, we find \( \alpha \), defined as the value \( \theta_L \) such that \( \theta_L > \theta^{R*} \) and \( \pi^*_F - \pi^*_L = \pi^*_F - \pi_R(\theta_L) = 0 \). Note that: \( \pi^*_F - \pi^*_L = 0 \) implies that:

\[
\theta^{R*}_L \Delta q - \theta_L \left[\Delta q \bar{b} + \eta c + \tau - \xi\right] + \eta \left[\frac{1}{\Delta q} (\Delta q \bar{b} + \xi - c)^2 + \bar{b} (\tau + \eta c - \xi)\right] = 0. \]

This is:

\[
\theta^{R*}_L = \frac{\left(\Delta q \bar{b} + \eta c + \tau - \xi\right) + \sqrt{\left(\Delta q \bar{b} + \eta c + \tau - \xi\right)^2 - 4\Delta q \bar{b}(\tau + \eta c - \xi)}}{2\Delta q} \]

Since \( \alpha > \theta^{R*} \), we eliminate the negative root: \( \alpha \) is the positive root. Finally, we show that \( \alpha > \theta^{F*} \). Recall that: (i) \( \theta^{F*} \) maximizes \( \pi^*_F \), (ii) \( \theta^{R*} \) maximizes \( \pi^*_R \), (iii) for any \( z, \pi_R(z) > \pi_F(z) \) and (iv) \( \theta^{R*} < \min\{\alpha, \theta^{F*}\} \). Assume that \( \alpha < \theta^{F*} \). Then there exists a \( z \), such that \( \alpha < z < \theta^{F*} \) and \( \pi_R(z) = \pi_F(z) \). This contradicts (iii).

21
Section 3.3: Analysis of the objective function.

The value of the objective function is:
\[
OF(\theta_L) = \begin{cases} 
\frac{2\theta^2}{2} - \frac{\theta^2}{2} - \Delta q\theta^R - \frac{1}{\eta}(\Delta q - \Delta q^L) + \frac{2}{\eta}((1 - \eta) \cdot (\Delta q_{\theta L} - \frac{\theta}{2}) + (1 + \eta) \cdot v(\bar{v} - \theta^R) - \frac{1}{\eta} \cdot \eta \cdot \eta & \text{if } \theta_L < \theta^R \\
\frac{2\theta^2}{2} - \frac{\theta^2}{2} - \Delta q\theta^R - \frac{1}{\eta}(\Delta q - \Delta q^L) + \frac{2}{\eta}((1 - \eta) \cdot (\Delta q_{\theta L} - \frac{\theta}{2}) + (1 + \eta) \cdot v(\bar{v} - \theta^R) - \frac{1}{\eta} \cdot \eta \cdot \eta & \text{if } \theta_L < \alpha \\
\frac{2\theta^2}{2} - \frac{\theta^2}{2} - \Delta q\theta^R - \frac{1}{\eta}(\Delta q - \Delta q^L) + \frac{2}{\eta}((1 - \eta) \cdot (\Delta q_{\theta L} - \frac{\theta}{2}) + (1 + \eta) \cdot v(\bar{v} - \theta^R) - \frac{1}{\eta} \cdot \eta \cdot \eta & \text{if } \theta_L > \alpha 
\end{cases}
\]

Shape of the objective function.

Note that \(\theta^F\) and \(\theta^R\) do not depend on \(\theta_L\). As a consequence, \(OF_1\) is constant in \(\theta_L\) and \(OF_3\) is increasing in \(\theta_L\).

We now study the shape of \(OF_2\). Note that:
\[
\frac{\partial OF_2}{\partial \theta_L} = -\Delta q\theta + \frac{1}{\eta}((1 - \eta)(\Delta \theta - \tau) - \frac{1 - \eta}{\eta} \Delta q (\bar{v} - \theta_L) - v).
\]

Thus, \(\frac{\partial OF_2}{\partial \theta_L} = 0\) if \(\theta_L = \theta_L^{MIN} = (1 - \eta)(\Delta \theta - \tau + v + \Delta q)/\Delta q\).

Note also that:
\[
\frac{\partial^2 OF_2}{\partial \theta_L^2} = -\Delta q + \frac{1 - \eta}{\eta} \Delta q < 0.
\]

Therefore:

a. If \(\eta < \frac{2}{3}\), \(\frac{\partial^2 OF_2}{\partial \theta_L^2} > 0\) and \(\theta_L^{MIN} \in (\theta^R, \alpha)\) is a minimum for \(OF_2\).

b. If \(\eta > \frac{2}{3}\), \(\frac{\partial^2 OF_2}{\partial \theta_L^2} < 0\) and \(\frac{\partial^2 OF_2}{\partial \theta_L^2} < 0\). \(OF_2\) is decreasing and convex in \(\theta_L \in (\theta^R, \alpha)\). To see this, note that since \(\eta > \frac{2}{3}\) and \(\Delta \bar{v} - \bar{c} > 0\) we have that \(\frac{2 - 3\eta}{\eta} \Delta q\theta_L - \frac{1 - \eta}{\eta}(\Delta \bar{v} - \bar{c}) < 0\), which implies that \(\frac{1 - \eta}{\eta} \Delta q\theta_L + \frac{1 - \eta}{\eta} \Delta q\theta_L - \frac{1 - \eta}{\eta} \Delta q\theta_L < 0\), i.e., \(\frac{\partial^2 OF_2}{\partial \theta_L^2} < 0\).

Finally, we note that \(OF_1 = OF_2(\theta^R)\), i.e., \(OF\) is continuous at \(\theta^R\). However, \(OF\) has a discontinuity at \(\alpha\).

Lemma 4. The following expression is the difference in payoffs between listing and not listing:
\[
\frac{1}{\Delta q} \left(\left(\theta^F\right)^2 - \left(\theta^R\right)^2\right) + v(\theta^F - \theta^R) - \left(\bar{v} - \theta^R\right) \left(\frac{1 - \eta}{\eta} \cdot \frac{\Delta q\theta^R - \Delta q\theta^R - \tau + \bar{c}}{\Delta q\theta^R - \tau + \bar{c}}\right)
\]

which after some algebra we can rewrite as:
\[
\frac{1}{\Delta q} \cdot [(c(1 - \eta) - \tau)(\Delta \bar{v} + 2v) + c^2/(2(1 + \eta) - 3/2 \cdot (c\eta + \tau)^2 - \frac{1}{2}(\Delta q\bar{v} + c - \tau)^2 + (\Delta q\bar{v} + c)^2 - \bar{c}(c(1 - \eta) - \tau))]
\]

Then:
\[
\frac{\partial OF_1(\theta_L) - OF_2^N}{\partial c} = \frac{1}{\Delta q} \cdot [(1 - \eta) \cdot \Delta \bar{v} + 2v) + c(1 + \eta - 3\eta^2) - 3\eta -
\]

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\[(1 - \eta) \frac{c}{\eta} \]  
\[= \frac{1}{\eta} \left[(1 - \eta) \left(\Delta q \bar{\eta} + 2v + c - \zeta\right) + 3\eta \cdot ((1 - \eta) c - \tau)\right] > 0.\]

\[\frac{\partial OF_1(\theta_L)}{\partial \eta} - \frac{\partial OF^{NL}}{\partial \eta} = -\frac{1}{\eta} \left(\Delta q \bar{\eta} + \zeta - \tau\right) + (\Delta q \bar{\eta} + c) - (c(1 - \eta) - \tau) = \]
\[-\frac{1}{\eta}\left[(\Delta q \bar{\eta} + \zeta)(1 - \eta) - (c(1 - \eta) - \tau)\right] \leq 0, \text{ since } \tau < (1 - \eta) \cdot (\Delta q \bar{\eta} + \zeta). \]

**Lemma 5.**

Note that the denominator of \(OF_1(\theta_L) - OF^{NL}\) is increasing in \(\Delta q\). Define the numerator of \(OF_1(\theta_L) - OF^{NL}\) as: \(Num(\Delta q \bar{\eta}) = (c(1 - \eta) - \tau)(\Delta q \bar{\eta} + 2v) + c^2/2(1 + 2\eta) - 3/2 \cdot (c\eta + \tau)^2 - \]
\[\frac{1}{\eta}(\Delta q \bar{\eta} + \zeta)^2 + (\Delta q \bar{\eta} + c)^2 - \zeta(c(1 - \eta) - \tau)).\]

Changes in \(\Delta q\) and \(\bar{\eta}\) have an identical effect on \(Num(\Delta q \bar{\eta})\). If \(Num(\Delta q \bar{\eta})\) is decreasing in \(\Delta q \bar{\eta}\), then \(OF_1(\theta_L) - OF^{NL}\) is decreasing in \(\Delta q\) and \(\bar{\eta}\). If \(Num(\Delta q \bar{\eta})\) is increasing in \(\bar{\eta}\) then \(OF_1(\theta_L) - OF^{NL}\) is increasing in \(\bar{\eta}\). We check under which conditions \(Num(\Delta q \bar{\eta})\) is decreasing in \(\Delta q \bar{\eta}\).

\[\frac{\partial Num(\Delta q \bar{\eta})}{\partial (\Delta q \bar{\eta})} = (c(1 - \eta) - \tau) - \frac{2}{\eta}(\Delta q \bar{\eta} + \zeta - \tau) + 2(\Delta q \bar{\eta} + c) = \]
\[= \frac{1}{\eta}\left[c(1 - \eta) - \tau \right] \leq 0. \text{ If this last expression is negative, then } Num(\Delta q \bar{\eta}) \text{ is decreasing in } \Delta q \bar{\eta}. \]

If this last expression is negative, then \(Num(\Delta q \bar{\eta})\) is decreasing in \(\Delta q \bar{\eta}\). However, note that \(\frac{\partial Num(\Delta q \bar{\eta})}{\partial (\Delta q \bar{\eta})}\) can have either a negative or a positive sign. For example, if \(\eta = 0\), then \(\frac{\partial Num(\Delta q \bar{\eta})}{\partial (\Delta q \bar{\eta})} \rightarrow \lim_{\eta \to 0} \left[\frac{1}{\eta}(-2\Delta q \bar{\eta} - 2\zeta - \tau)\right] < 0\) and if \(\eta = 1\), then \(\frac{\partial Num(\Delta q \bar{\eta})}{\partial (\Delta q \bar{\eta})} = -\tau + 2\eta > 0. \) Indeed \(\frac{\partial Num(\Delta q \bar{\eta})}{\partial (\Delta q \bar{\eta})}\) is increasing in \(\eta:\)

\[\frac{\partial}{\partial \eta} \left(\frac{\partial Num(\Delta q \bar{\eta})}{\partial (\Delta q \bar{\eta})}\right) = -c + 2(\Delta q \bar{\eta} + \zeta - \tau) - \eta^2/\eta^2c > 0.\]

Hence, for small values of \(\eta\) we obtain that \(\frac{\partial Num(\Delta q \bar{\eta})}{\partial (\Delta q \bar{\eta})} < 0\) and for large values of \(\eta\), \(\frac{\partial Num(\Delta q \bar{\eta})}{\partial (\Delta q \bar{\eta})} > 0.\)

**Lemma 6.**

Here:

\[OF_1(\theta_L) - OF^{NL} = \]
\[= \frac{1}{\eta} \left[c(1 - \eta)(\Delta q \bar{\eta}) + c^2/2(1 + 2\eta) - 3/2 \cdot (c\eta + \tau)^2 - \frac{1}{\eta}(\Delta q \bar{\eta})^2 + (\Delta q \bar{\eta})^2\right] = \]
\[= \frac{1}{\eta} \left[-(1 - \eta)(\Delta q \bar{\eta})^2 + (1 - \eta)\eta \Delta q \bar{\eta} c + c^2/2(\eta + 2\eta^2 - 3\eta^3)\right] = \]
\[= \frac{1 - \eta}{\eta} \left[-(\Delta q \bar{\eta})^2 + \eta \Delta q \bar{\eta} c + c^2/2(\eta + 3\eta^2)\right].\]

Hence, if \(-(\Delta q \bar{\eta})^2 + \eta \Delta q \bar{\eta} c + c^2/2(\eta + 3\eta^2) < 0\) then \(OF_1(\theta_L) - OF^{NL} < 0.\)

Studying the last expression we obtain that \(OF_1(\theta_L) - OF^{NL} < 0\) is negative
if $\Delta q\bar{\theta}$ exceeds $c/2(\eta + \sqrt{\eta^2 + 2\eta})$ (the positive root of the polynomial). Note that $c/2(\eta + \sqrt{\eta^2 + 2\eta})$ is increasing in $\eta$. Hence, $OF_1(\theta_L) - OF^{NL}$ is least likely to be negative when $\eta = 1$. Note that in this case $c/2(\eta + \sqrt{\eta^2 + 2\eta}) = 2c$. In conclusion, if $\Delta q\bar{\theta} > 2c = \max_{\eta}(c/2(\eta + \sqrt{\eta^2 + 2\eta}))$, $0 \leq \eta < 1$, then $\Delta q\bar{\theta} > c/2(\eta + \sqrt{\eta^2 + 2\eta})$ for all $\eta, 0 \leq \eta < 1$, and we can conclude that $OF_1(\theta_L) - OF^{NL} < 0$.

Section 4: Price commitment. Local maximization of OF with respect to $\theta_L$.

We start by finding the value of $\theta_L$ that maximizes each region of $OF$.

1. $\theta_L < \theta^R$. Here: $OF_1 = q\bar{\theta} - \frac{\eta^2}{2} - \Delta q\frac{\eta^2}{2} - (1 - \eta) P(\bar{\theta} - \theta^R)$, and $\frac{\partial OF_1}{\partial \theta_L} = 0$.

Therefore, the agency is indifferent between any $\theta_L$, for any $\theta_L \in (\theta, \theta^R)$.

2. $\theta^R < \theta_L < \theta^F$. Here: $\frac{\partial OF_2}{\partial \theta_L} = -\Delta q\theta_L + (1 - \eta) P = 0 \Rightarrow \theta^* = \frac{(1-\eta)P}{\Delta q}$ and $\frac{\partial^2 OF_2}{\partial \theta_L^2} = -\Delta q < 0$. Hence: $OF_2^* = q\bar{\theta} - \frac{\eta^2}{2} - \Delta q\frac{\eta^2}{2} - (1 - \eta) P(\bar{\theta} - \theta^*_2)$.

Note that $\theta^F > \theta^* > \theta^R$ implies that $\eta P < (1 - \eta) P < P$. If $\eta < 1/2$, we have that $\theta^F > \theta^* > \theta^R$ but if $\eta > 1/2$, we have that $\theta^F > \theta^R > \theta^*$, which invalidates $OF_2^*$ as a solution as consumer $\theta^*$ does not purchase the drug.

3. $\theta^F < \theta_L$. Here: $\frac{\partial OF_3}{\partial \theta_L} = (1 - \eta) P > 0 \Rightarrow \theta^*_3 = \bar{\theta}$ and $OF_3^* = q\bar{\theta} - \frac{\eta^2}{2} - \Delta q\frac{\eta^2}{2}$.

Proposition 7.

1. $\eta > 1/2$. We compare $OF_1^*$ and $OF_3^*$:

$$OF_1^* - OF_3^* = \left(\theta^F - \theta^R\right)^2 - \left(\theta^R - \theta^R\right)^2 - P(1 - \eta)\left(\bar{\theta} - \theta^R\right) =$$

$$\left(\theta^F - \theta^R\right)^2 - \left(\theta^F + \theta^F\right)\frac{\Delta q}{2} - P(1 - \eta)\left(\bar{\theta} - \theta^R\right) = \frac{(1-\eta)P}{\Delta q} - \frac{(1+3\eta)P}{2} - \Delta q\bar{\theta} - \eta P).$$

Note that $(1-\eta)P(\frac{(1+3\eta)P}{2} - \Delta q\bar{\theta}) > 0$ iff $P > 2\Delta q\bar{\theta} > (1+3\eta)$. Recall that $\eta P < \Delta q\bar{\theta}$. This condition is compatible with $P > \frac{2}{(1+3\eta)}\Delta q\bar{\theta}$ as $2\eta - 3\eta < 1$.

2. $\eta < 1/2$. We compare $OF_2^*$ and $OF_3^*$:

$$OF_2^* - OF_3^* = \frac{\Delta q}{2} \left(\theta^F - \theta^F\right)^2 - (1 - \eta) P(\bar{\theta} - \theta^*) =$$

$$\frac{\Delta q}{2} \left(\theta^F - \theta^F\right)^2 - (1 - \eta) P(\bar{\theta} - \theta^*) =$$

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\[
\frac{P}{\Delta q} \left( \frac{1}{2} (2 - \eta) P \eta - (1 - \eta) \left( \Delta q \bar{\theta} - (1 - \eta) P \right) \right) > 0.
\]

Note that \( OF^{2*} - OF^{3*} > 0 \) iff \( P > \frac{(1 - \eta) \Delta q \bar{\theta}^2}{2 + \eta^2 - 2 \eta} \).

Recall that \( \eta P < \Delta q \bar{\theta} \). This last condition is compatible with \( P > \frac{(1 - \eta) \Delta q \bar{\theta}^2}{2 + \eta^2 - 2 \eta} \) since \( \eta < \frac{1}{2} \) implies that \( \frac{2(1 - \eta)}{2 + \eta^2 - 2 \eta} < \frac{1}{\eta} \).

\[\Box\]

**Proposition 8.**

If the drug is listed for reimbursement the firm’s profits are: \( (P - c)(\bar{\theta} - \theta^R) = (P - c)(\bar{\theta} - \frac{\eta P}{\Delta q}) \). These profits are maximized when \( P^* = \frac{\Delta q \bar{\theta} + \eta P}{2 \eta} \). Their value is \( \frac{1}{4 \eta \Delta q} (\Delta q \bar{\theta} - \eta c)^2 \). This value exceeds the maximum value of the profits achieved with no reimbursement: \( \frac{1}{(1 - \eta)(2 + \eta^2 - 2 \eta)} (\Delta q \bar{\theta} - c)^2 \). Moreover, \( P^* = \frac{\Delta q \bar{\theta} + \eta P}{2 \eta} > \frac{2 \Delta q \bar{\theta}}{1 + \eta} \), since \( \Delta q \bar{\theta} (1 - \eta) + \eta (1 + 3 \eta) c > 0 \). This implies Proposition 8.

\[\Box\]

**Proposition 9.**

Recall that for \( \eta < \frac{1}{2} \), if \( P > P_{LIM} \), the agency sets \( \theta_L^* = \frac{(1 - \eta) P}{\Delta q} \), but if \( P < P_{LIM} \), the agency sets \( \theta_L = \bar{\theta} \). For \( P^+ \) to encourage listing, it must be that \( P^+ > P_{LIM} \), this is that \( c > \Delta q \bar{\theta} \frac{2 - 6 \eta + 3 \eta^2}{(1 - \eta)(2 + \eta^2 - 2 \eta)} \). If this last inequality holds, Proposition 9.1 follows.

However, if \( c < \Delta q \bar{\theta} \frac{2 - 6 \eta + 3 \eta^2}{(1 - \eta)(2 + \eta^2 - 2 \eta)} \), \( P^+ \) does not induce the agency to list the drug and the firm’s optimal choice will either be \( P_{LIM} \) (which induces listing) or \( \frac{1}{2} (\Delta q \bar{\theta} + c) \), which maximizes the profit with no listing (\( \pi_R \)). Proposition 9.2 provides the condition under which \( P_{LIM} \) is best. The following steps yield such condition.

**Step 1:** We define the profits of no listing (\( \theta_L = \bar{\theta} \)) and of listing (\( \theta_L = \theta_L^* \)) as a function of the indifferent consumer \( I \). With no listing \( I = \bar{\theta} \), \( \pi^NL[I] = \Delta q I - c (\bar{\theta} - I) \). With listing, \( I = \theta_L^* = (1 - \eta) P / \Delta q \), so: \( \pi^L[I] = (P - c)(\bar{\theta} - I) = \frac{1}{1 - \eta} (\Delta q I - (1 - \eta) c) (\bar{\theta} - I) \).

**Step 2:** We proof that for any fixed \( I \in [0, \bar{\theta}] \), \( \pi^L[I] > \pi^NL[I] \).\(^{37}\) Note that: \( \pi^L - \pi^NL = \left( \frac{1}{1 - \eta} (\Delta q I - (1 - \eta) c) - (\Delta q I - c) \right) (\bar{\theta} - I) \). Hence if \( \bar{\theta} > I \), then \( \pi^L - \pi^NL > 0 \).

**Step 3:** We define \( z \) as the value of \( \theta \) that induces listing, this is \( z \in (\theta^+, \bar{\theta}) \) such that \( \pi^L(z) = \pi^NL + \frac{1}{\Delta q^2} (\Delta q \bar{\theta} - c)^2 \). Therefore, \( z \) is such that: \( \left( \frac{\Delta q \bar{\theta}}{1 - \eta} - c \right) (\bar{\theta} - z) = \frac{(\Delta q \bar{\theta} - c)^2}{4 \Delta q} \). This equation has two roots:

\[
\frac{\Delta q \bar{\theta} - (1 - \eta) c}{2 \Delta q} \pm \frac{\sqrt{\left( \frac{\Delta q \bar{\theta} - c}{2 \Delta q} \right)^2 - \left( \frac{\eta \Delta q \bar{\theta}^2 - (1 - \eta) c}{2 \Delta q} \right)}}{2 \Delta q} = \theta^+ \pm \sqrt{\left( \frac{\Delta q \bar{\theta} - c}{2 \Delta q} \right)^2 - \left( \frac{\eta \Delta q \bar{\theta}^2 - (1 - \eta) c}{2 \Delta q} \right)}.
\]

\(^{37}\)If \( \bar{\theta} = I \), then \( \pi^NL[I] = \pi^L[I] \).

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Note that the smallest root is smaller than $\theta^+$, therefore it can not induce listing. Hence, $z = \theta^+ + \sqrt{\frac{\eta((\Delta q\theta)^2-c^2(1-\eta))}{2\Delta q}}$. Note for any $\theta > z$, $\pi^L < \pi^{NL^*}$ and for any $\theta < z$, $\pi^L > \pi^{NL^*}$.

**Step 4:** If the price is $P_{\text{LIM}}$, the indifferent consumer is $\theta_{\text{LIM}} = \theta_L - P_{\text{LIM}} = \theta_L - \frac{\Delta q}{\Delta q^2} 2(1-\eta)^2\eta - 2\eta - (1-\eta)^2\Delta q$. Note that $\theta_{\text{LIM}} > \theta^+$ since $P_{\text{LIM}} > P^+ = \frac{c}{2} + \sqrt{\frac{\Delta q}{2(1-\eta)}}$.

**Step 5:** Given steps 3 and 4 we can conclude that: if $\theta_{\text{LIM}} < z$, then, the firm will prefer to induce listing and $P_{\text{LIM}}$ will be the global maximum. This is the condition stated in proposition 9.2.

**Corollary 10.** Note that: $\theta_{\text{LIM}} < z \iff \frac{2(1-\eta)^2\theta - \theta}{2\eta^2-2\eta} < \frac{c(1-\eta)^2}{2\Delta q} + \frac{1}{2\Delta q} \sqrt{\frac{\eta((\Delta q\theta)^2-(1-\eta))c^2}{2\Delta q}}$,

which can be rewritten as: $\frac{\Delta q\theta}{2(1-\eta)^2\eta + (1-\eta)^2} - \frac{c}{2} < \frac{1}{2\Delta q} \sqrt{\frac{\eta((\Delta q\theta)^2-(1-\eta))c^2}{2\Delta q}}$.

The above inequality can revert sign depending on the value of $\eta$. For example if $c = 0$, the condition simplifies to $2 + 3\eta^2 - 6\eta - (2 + \eta^2 - 2\eta) \sqrt{\eta}< 0$. This is satisfied when $\eta > 0.2$.

**Section 5.**

We first prove that $\frac{2\Delta q\theta}{(\eta + \sqrt{\eta^2+2\eta})} > c^*$. This implies that:

$2(1-\eta)(2 + \eta^2 - 2\eta) - (\eta + \sqrt{\eta^2+2\eta})(2 + 3\eta^2 - 6\eta) > 0$

This expression holds for $\eta = 0$ and $\eta = 1/2$. Below, we provide a numerical evaluation of the expression for $0 < \eta < 1/2$:

![Graph](image)

The expression has a minimum at $\eta = 0.326$. We evaluate the expression in this point to obtain its positive value 1.4134.
Section 5, Table 1: Proof of cases c, d and e

Note that in all these cases $\eta < \frac{1}{2}$.

Case c (comparison of the outcomes with no price commitment ($\theta^R*$) and price commitment ($\theta^+$)).

Note that $P^+ > P_R$ and $\theta^R* < \theta^+$. This implies that the demand is larger with no price commitment and that the price is larger with commitment.

Profit Comparison:
Since $\pi^+ = \frac{1-\eta}{\Delta q^+} (\Delta q^+ - c)^2$ and $\pi^R* = \frac{1-\eta}{4\eta\Delta q^*} (\Delta q^* - \eta c)^2$: $\pi^R* > \pi^+ \Leftrightarrow \frac{1}{\Delta q^+} (\Delta q^* - \eta c)^2 > \frac{1}{\Delta q^*} (\Delta q^+ - (1-\eta)c)^2$. This expression holds iff $(1-2\eta)((\Delta q^* - \eta (1-\eta)c) > 0$. Since $\eta < \frac{1}{2}$ and $\Delta q^* > c$ we conclude that $\pi^R* > \pi^+$, i.e.: $\pi^{NPC} > \pi^{PC}$.

Agency's objective function comparison:
Note that $OF^{NPC} = \frac{\eta^2}{2} - \frac{\eta^2}{2} - \Delta q^* \frac{\theta^R*}{2} - (1-\eta) P_R (\overline{\theta} - \theta^R*)$ and
$OF^{PC} = \frac{\eta^2}{2} - \frac{\eta^2}{2} - \Delta q^* \frac{\theta^+}{2} - (1-\eta) P^+ (\overline{\theta} - \theta^+)$.

Hence:

$OF^{PC} - OF^{NPC} = \frac{\Delta q}{\eta \Delta q^*} \left( (\Delta q^* - \eta c)^2 - (\Delta q^+ - (1-\eta)c) \right) + (1-\eta) \left( \frac{\Delta q^* + \eta c}{\eta \Delta q^*} - \frac{\Delta q^+ + (1-\eta)c}{\eta \Delta q^+} \right) \left( \frac{\Delta q^* - (1-\eta)c}{\eta} \right)$.

Note that if $\eta \to 0$ then $OF^{PC} - OF^{NPC} \to +\infty$. Recall that if $\eta \to 1/2$, $OF^{PC} - OF^{NPC} \to 0$. We now prove that $OF^{PC} - OF^{NPC} > 0$ for $0 < \eta < 1$. By simplifying the expression we obtain: $\Delta q^* (1-2\eta) \frac{\Delta q^* - \eta c}{\eta} + 1 - 4\eta (1-\eta) \frac{c^2}{2}$. To prove this, take the derivative of this expression with respect to $c$:
$-\Delta q^* (1-2\eta) + 1 - 4\eta (1-\eta)c$. Since $0 < \eta < 1/2$ and $\Delta q^* > c$, that this derivative is negative. Hence the expression is least likely to hold for large $c$. A sufficient condition would be if it held for $c = \Delta q^*$. In this case $OF^{PC} - OF^{NPC}$ becomes: $c^2 (1-2\eta) \frac{1-\eta}{\eta} + 1 - 4\eta (1-\eta) \frac{c^2}{2}$. It is positive whenever $(1-2\eta) \frac{1-\eta}{\eta} + 1 - 4\eta (1-\eta) \frac{c^2}{2} > 0$. This is $(1-2\eta) (1-\eta) + (1 - 4\eta (1-\eta) \eta > 0$ or $0 < 4\eta^3 - 5\eta + 2$. This expression is positive for $0 < \eta < 1/2$. Therefore, $OF^{PC} > OF^{NPC}$.

Case d. (Comparison of the outcomes with no price commitment ($\theta^{F*}$) and price commitment ($\theta^+$))

Profit Comparison:

\footnote{If $\eta = 1/2$, $P^+ = P_R = \frac{1}{2} \Delta q^*}$ and the two outcomes coincide.

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Since $\pi^+ = \frac{1-\eta}{4\Delta q^2} \left( \Delta q\theta - c \right)^2$ and $\pi^{F*} = \frac{1-\eta}{4\Delta q^2} \left( \Delta q\theta - c \right)^2$ we obtain that: $\pi^+ - \\
\pi^{F*} = \frac{1-\eta}{4\Delta q^2} \left( \Delta q\theta - c \right)^2 > 0$ which clearly holds as: $(\Delta q\theta - (1-\eta)c)^2 > (1-\eta) \left( \Delta q\theta - c \right)^2$

Agency’s objective function comparison:

Note that $\text{OF}_{\text{NPC}} = \frac{\theta^2}{2} - \frac{\theta^2}{2} - \Delta q\theta^2$ and $\text{OF}_{\text{PC}} = \frac{\theta^2}{2} - \frac{\theta^2}{2} - \Delta q\theta^2 - \\
(1-\eta) P^+ (\theta - \theta^+)$. Hence, $\text{OF}_{\text{NPC}} - \text{OF}_{\text{PC}} = \frac{\Delta q}{2} \left( \theta^2 - \theta^{F*} \right) + (1-\eta) P^+ (\theta - \theta^+)$. 

Substituting the different variables and simplifying the expression we get:

$\text{OF}_{\text{NPC}} - \text{OF}_{\text{PC}} = \frac{1}{4\Delta q} \left( \frac{\Delta q}{2} (\eta - 2) - \eta \Delta q\theta\bar{\theta} + (\Delta q\theta)^2 - (1-\eta)^2 c^2 \right) = \\
\frac{1}{4\Delta q} \left( \eta - \frac{3}{2} \eta^2 - 1 \right) c^2 - \eta \Delta q\theta\bar{\theta} + (\Delta q\theta)^2$

Hence $\text{OF}_{\text{NPC}} - \text{OF}_{\text{PC}} > 0$ iff $(\Delta q\theta)^2 - \eta \Delta q\theta\bar{\theta} - c^2 (1-\eta) (1-\frac{1}{2}\eta) > 0$.

Note that $\frac{\partial c}{\partial \text{OF}_{\text{NPC}} - \text{OF}_{\text{PC}}} = -\eta \Delta q\theta\bar{\theta} - 2c (1-\eta (1-\frac{1}{2}\eta)) < 0$.

Hence, $\text{OF}_{\text{NPC}} - \text{OF}_{\text{PC}}$ is decreasing in $c$ if $0 \leq \eta \leq 1/2$ and has a maximum at $c = 0$, where the expression achieves a positive value of $(\Delta q\theta)^2$. The positive root of $\text{OF}_{\text{NPC}} - \text{OF}_{\text{PC}}$ is $c^+ = \frac{1}{2\eta + \eta^2 + 2} \left( -\Delta q\theta\eta + \sqrt{4(\Delta q\theta)^2 - 4(\Delta q\theta)^2 \eta + 3(\Delta q\theta)^2 \eta^2} \right)$. 

This implies that if $c < c^+$ then $\text{OF}_{\text{NPC}} - \text{OF}_{\text{PC}} > 0$ and otherwise $\text{OF}_{\text{NPC}} - \text{OF}_{\text{PC}} < 0$.

Recall that in case (d) $\frac{\Delta q}{(1-\eta)(2+\eta^2-2\eta)} \leq c \leq \frac{2\Delta q}{\eta^2 + 2\eta}$, we check the relative position of $c^+$ with respect to these two values:

1. $\frac{\Delta q}{(1-\eta)(2+\eta^2-2\eta)} < c^+$. This is that:

$\frac{\Delta q}{(1-\eta)(2+\eta^2-2\eta)} < \frac{1}{2\eta + \eta^2 + 2} \left( -\Delta q\theta\eta + \sqrt{4(\Delta q\theta)^2 - 4(\Delta q\theta)^2 \eta + 3(\Delta q\theta)^2 \eta^2} \right) \rightarrow \\
(2 + 3\eta^2 - 6\eta) < (1-\eta) \left( -\eta + \sqrt{4 - 4\eta + 3\eta^2} \right) \rightarrow \\
(2 + 3\eta^2 - 6\eta) + \eta(1-\eta) < (1-\eta) \sqrt{4 - 4\eta + 3\eta^2} \rightarrow \\
2\eta^2 - 5\eta + 2 < (1-\eta) \sqrt{4 - 4\eta + 3\eta^2} \rightarrow \\
(2\eta^2 - 5\eta + 2)^2 - (1-\eta)^2 (4 - 4\eta + 3\eta^2) = 18\eta^2 - 8\eta - 10\eta^3 + \eta^4 < 0 \rightarrow \\
18\eta - 8 - 10\eta^2 + \eta^3 < 0.$

If $\eta = 0$ the expression holds. If $\eta = 1/2$, then the expression is $-1.375$.

Plotting the expression we find that $18\eta - 8 - 10\eta^2 + \eta^3$ is:
2. $c^+ < \frac{2\Delta q\eta}{\eta + \sqrt{7\eta^2 + 2\eta}}$. This is that:

\[-\frac{1}{2\eta + \eta + 2} \left( -\Delta q\bar{\eta} \eta + \sqrt{4(\Delta q\bar{\eta})^2 - 4(\Delta q\bar{\eta})^2\eta + 3(\Delta q\bar{\eta})^2\eta^2} \right) < \frac{2\Delta q\eta}{\eta + \sqrt{7\eta^2 + 2\eta}} \rightarrow \]

\[-\frac{1}{2\eta + \eta + 2} \left( -\eta + \sqrt{4 - 4\eta + 3\eta^2} \right) < \frac{2}{\eta + \sqrt{7\eta^2 + 2\eta}} \left( \frac{\eta + \sqrt{7\eta^2 + 2\eta}}{-\eta + \sqrt{4 - 4\eta + 3\eta^2}} \right) \rightarrow \]

\((\eta + \sqrt{7\eta^2 + 2\eta}) \left(-\eta + \sqrt{4 - 4\eta + 3\eta^2} \right) - 2(-2\eta + \eta^2 + 2) < 0 \rightarrow \]

\((\eta + \sqrt{7\eta^2 + 2\eta}) \left(-\eta + \sqrt{4 - 4\eta + 3\eta^2} \right) - 2(-2\eta + \eta^2 + 2) < 0 \]

Note that if 

\((\eta + \sqrt{7\eta^2 + 2\eta}) \left(-\eta + \sqrt{4 - 4\eta + 3\eta^2} \right) - 2(-2\eta + \eta^2 + 2) = 0 \]

then \( \eta = -2 \). Indeed by plotting this expression we find that

\((\eta + \sqrt{7\eta^2 + 2\eta}) \left(-\eta + \sqrt{4 - 4\eta + 3\eta^2} \right) - 2(-2\eta + \eta^2 + 2) \) is:
If \( \eta = 0 \) then the expression is \(-4\) and if \( \eta = 1/2 \) the expression is \(-2\).

1684 \times 10^{-19}

In conclusion we obtain the following: If \( c \in \{ \min(0, \frac{\Delta q(2+3\eta^2-6\eta)}{(1-\eta)(2+\eta^2-2\eta)})), c^+ \} \) then \( OF_{NPC} - OF_{PC} > 0 \). If \( c \in \{ c^+, \frac{2\Delta q}{\eta+\sqrt{7\eta^2+2\eta}} \} \) then \( OF_{NPC} - OF_{PC} < 0 \).

**Case e1 (Comparison of the outcomes with no price commitment (\( \theta^{*} \)) and price commitment (\( \theta^{LIM} \))**

**Profit Comparison:** See the proof of lemma 10 for reference.

**Agency’s objective function comparison:** Note that \( OF_{NPC} = OF_{NL} = q \frac{\theta^2}{2} - \frac{\eta^2}{2} - \Delta q \frac{\theta^{F'}^2}{2} \) and \( OF_{PC} = q \frac{\overline{\theta}^2}{2} - \frac{\eta^2}{2} - \Delta q \frac{\theta^{LIM^2}}{2} - (1-\eta) P^{LIM} (\overline{\theta} - \theta^{LIM}) \).

Recall that in case d we have proven that for \( c < c^+ \) then \( OF_{NL} - OF_{+} > 0 \).

Case e1 is relevant when \( 0 < c < \frac{\Delta q(2+3\eta^2-6\eta)}{(1-\eta)(2+\eta^2-2\eta)} \) (\( < c^+ \)). Hence in this case we know that \( OF_{NL} - OF_{+} > 0 \). We now prove that \( OF_{+} - OF_{LIM} > 0 \) to conclude that in case d \( OF_{NL} - OF_{LIM} > 0 \) this is that \( OF_{NPC} > OF_{PC} \).

**Proof that \( OF_{+} - OF_{LIM} > 0 \).** This entails proving that \( \frac{\partial OF_{+}(P)}{\partial P} < 0 \) where

\[
OF_{+} = q \frac{\overline{\theta}^2}{2} - \frac{\eta^2}{2} - \Delta q \frac{\theta^{+2}}{2} - (1-\eta) P (\overline{\theta} - \theta^{+}) \text{ and } \theta^{+} = \frac{(1-\eta)P}{\Delta q} \text{ hence: } OF_{+} = q \frac{\overline{\theta}^2}{2} - \frac{\eta^2}{2} - \Delta q \left( \frac{(1-\eta)P}{\Delta q} \right)^2 - (1-\eta) P (\overline{\theta} - \frac{(1-\eta)P}{\Delta q}) ; \frac{\partial OF_{+}}{\partial P} = -\Delta q \left( \frac{1-\eta}{\Delta q} \right)^2 P - (1-\eta) \left( \overline{\theta} - \frac{2(1-\eta)P}{\Delta q} \right) = -\Delta q \left( \frac{1-\eta}{\Delta q} \right)^2 P - (1-\eta) \overline{\theta} + \frac{2(1-\eta)^2}{\Delta q} P = \frac{1-\eta}{\Delta q} P - (1-\eta) \overline{\theta}.
\]

Hence \( \frac{\partial OF_{+}}{\partial P} < 0 \) if \( \frac{(1-\eta)P}{\Delta q} - \overline{\theta} < 0 \) this is \( (1-\eta)P < \Delta q \overline{\theta} \). This is satisfied as \( \theta^{+} = \frac{(1-\eta)P}{\Delta q} < \overline{\theta} \).
Figure 1: Demand function.
Figure 2: Profit function.
Figure 3: The \( OF \) function.

Low subsidies case
High subsidies case

Figure 4: The OF function.
Figure 5: The OF function with price commitment.
Figure 6: OF function with price commitment.
Figure 7: Listing with price commitment.
<table>
<thead>
<tr>
<th>Subsidy level</th>
<th>Cost level</th>
<th>Commitment</th>
<th>Non commitment</th>
<th>Comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta &gt; 1/2$</td>
<td>(a) $c &gt; \frac{2,\Delta q,\bar{\theta}}{\eta + \sqrt{7}\eta^\prime + 2\eta}$</td>
<td>List, $\theta^{*}$</td>
<td>List, $\theta^{**}$</td>
<td>$\pi^{RC} = \pi^{SNC}$ $OF^{RC} = OF^{SNC}$</td>
</tr>
<tr>
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<td>(b) $c &lt; \frac{2,\Delta q,\bar{\theta}}{\eta + \sqrt{7}\eta^\prime + 2\eta}$</td>
<td>List, $\theta^{*}$</td>
<td>Not list, $\theta^{**}$</td>
<td>$\pi^{RC} &gt; \pi^{SNC}$ $OF^{RC} &gt; OF^{SNC}$</td>
</tr>
<tr>
<td>$\eta &lt; 1/2$</td>
<td>(c) $c &gt; \frac{2,\Delta q,\bar{\theta}}{\eta + \sqrt{7}\eta^\prime + 2\eta}$</td>
<td>List, $\theta^{*}$</td>
<td>List, $\theta^{**}$</td>
<td>$\pi^{SNC} &gt; \pi^{RC}$ $OF^{SNC} &gt; OF^{RC}$</td>
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<td>(d) $c^{*} &lt; c &lt; \frac{2,\Delta q,\bar{\theta}}{\eta + \sqrt{7}\eta^\prime + 2\eta}$</td>
<td>List, $\theta^{*}$</td>
<td>Not list, $\theta^{**}$</td>
<td>$\pi^{PC} &gt; \pi^{NPC}$ for large $c$ $OF^{PC} &gt; OF^{NPC}$ for small $c$ $OF^{SNC} &gt; OF^{PC}$</td>
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<tr>
<td>(e1) $c = c^{*}$</td>
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<td>Not list, $\theta^{**}$</td>
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<td>(e2) $c &lt; c^{*}$</td>
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<td>Not list, $\theta^{**}$</td>
<td>$\pi^{SNC} = \pi^{RC}$ $OF^{SNC} = OF^{RC}$</td>
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</tr>
</tbody>
</table>

Figure 8: Table 1: Impact of price commitment on listing decision.