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Modeling Stock Pinning

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Abstract

The paper investigates the effect of hedging strategies on the so called pinning effect, i.e. the tendency of stock's prices to close near the strike price of heavily traded options as the expiration date nears. In the paper we extend the analysis of Avellaneda and Lipkin (2003) who propose an explanation of stock pinning in terms of delta hedging strategies for long option positions. We adopt a model introduced by Frey and Stremme (1997) and show that in this case pinning is driven by two effects: a hedging dependent drift term that pushes the stock price toward the strike price and a hedging dependent volatility term that constrains the stock price near the strike as it approaches it. Finally we show that pinning can be gnerated by dynamic hedging strategies under more realistic market conditions by simulating trading in a double auction model.

1 Introduction

Financial mathematics models are typically based on the assumption that markets are complete, frictionless and perfectly liquid, the last implying that investors can trade large volumes of stocks without affecting their prices. Each of these assumptions have been questioned in the literature. In particular a number of papers (Frey and Stremme (1997, 2000), Platen and Schweizer (1998), Schonbucher and Wilmott (2000)) have concentrated on the feedback effects of dynamic hedging strategies in illiquid markets. These papers focus on the impact of options' hedging strategies on the volatility of the underlying asset and relate the smile pattern of implied volatility to the lack of liquidity. In this paper we analyze the impact of option hedging strategies on both the drift and the volatility of the underline asset price and show that dynamic hedging can be responsible for the pinning effect, i.e. the tendency of stock's prices to close near the strike price of heavily traded options (in the same stock) as the expiration date nears.

A thorough analysis of the pinning effect has been provided by Ni et al. (2005) who analyze data from the Ivy DB dataset and from a second dataset obtained from the CBOE, from 1996 to 2002. The authors show that over this period, optionable stocks (i.e., stocks with listed options) close near the strike prices on expiration dates, both when the likely delta hedgers have net purchased option positions and net written option positions. There is no corresponding effect for non-optionable stocks.[1] Moreover, as the expiration date approaches, the pinning effect increases when hedgers have net long option positions, but it decreases when delta hedgers have net short option positions. Thus the authors conclude that when traders have net long positions delta hedging does contribute to the pinning. On the contrary, when traders have net short positions, the pinning effects is driven by stock manipulation. Evidence of pinning is provided also by Krishnan and Nelken (2001) and Avellaneda and Lipkin (2003).

Avellaneda and Lipkin (2003) introduce the first pinning model by suggesting that stock pinning can be induced by delta hedging long option positions. In our paper we extend their analysis following the approach of Frey and Sremme (1997, 2000). In our model pinning is the result of two combined effects: a drift term, driving the stock price towards the strike price of the option, and a volatility term that, by decreasing as the stock price approaches the strike price, confines the price close to the strike. Both models Frey and Stremme and Avellaneda and Lipkin, make a number of assumptions on the dynamics of prices, on continuous hedging and on the price impact shape. To model the price formation and the feedback mechanism in a more realistic framework, we study pinning in a microstructure

[1] The CBOE database used by Ni et al. provides information, for each transaction, on whether the two parties trading are market makers, public customers, or firm proprietary traders. By assuming that the public customers do not hedge their option portfolio Ni and al. infer if the hedgers overall net position is long or short.

model initially introduced by Marcus et al. (2003) (see also Farmer et al (2003) and Smith et al. (2004)). In the model, zero intelligence agents can submit both market and limit orders. The order flows are modeled as Poisson process. The original model has been expended by incorporating an hedger who rebalances his position discretely. We show that our model generates pinning consistently with the theoritical models and the experimental findings.

The remainder of this paper is organized as follow: in section 2 we revise the Frey and Stremme model (1998) and highlight the mechanisms in the model that can induce pinning. In this section we also compare our analysis to the one performed by Avellaneda and Lipkin (2003). In section 3 we describe the microstructure model and the results of our simulations.

2 Stock Pinning

Assume that the asset price behaviour is defined by a diffusion equation of the form,

$$dS(t) = \mu S(t)dt + \sigma S(t)dW(t), \tag{1}$$

where μ is a constant drift, σ a constant volatility, and $W(t)$ a standard Brownian motion. Under this assumption the value of a European vanilla call option is given by Black and Scholes formula,

$$C(S(t), T, K, \sigma, r) = S(t)N(d_1) - Ke^{-r(T-t)}N(d_2),$$

with

$$d_{1,2} = \frac{\log(\frac{S(t)}{K}) + (r \pm 1/2\sigma)^2(T-t)}{\sigma\sqrt{T-t}}.$$

where r is the risk free rate, K the option strike price and T the option maturity. $N(\cdot)$ is the normal cumulative distribution function. Moreover the amount of stock to hold in order to hedge a position is given by the option delta defined as $\Delta = \partial C/\partial S$. As time goes the amount of stocks to buy or sell to maintain a delta neutral is given by $d\Delta$.

Δ hedging strategies create a feedback effect on the stock price by inducing an additional drift term in the diffusion equation given by eq. (1) that becomes

$$dS(t) = \mu S(t)dt + n\hat{L}S(t)d\Delta(S,t) + \sigma S(t)dW(t). \tag{2}$$

In eq.(2) above $\hat{L}$ is a constant price elasticity and n is the open interest on the call option. Hence, $\hat{L}$ represents a linear impact of the hedgers on the stock process. The model further assumes that traders do not take into account feedbacks effects when rebalancing their portfolio. The hedging strategy is based on the assumption that the stock price evolves accordingly to the geometric brownian motion in eq.(1). In this case the Delta for a long call is $\Delta = -N(d_1)$.

The dynamics of $\Delta(S,t)$ can be derived by using Itô's Lemma which gives,

$$d\Delta(S,t) = \frac{\partial \Delta(S,t)}{\partial t}dt + \frac{\partial \Delta(S,t)}{\partial S}dS(t) + \frac{1}{2}\frac{\partial^2 \Delta(S,t)}{\partial S^2}\langle dS(t)\rangle. \tag{3}$$

where $\langle dS(t)\rangle$ denotes the quadratic variation of $S(t)$.

Avellaneda and Lipkin (2003) assume that the price dynamics is given by

$$dS(t) = n\hat{L}\,\frac{\partial \Delta}{\partial t}S(t)dt + \sigma S(t)dW(t). \tag{4}$$

This can be interprated as taking only the delta time decay term in eq.(3) above. The model generates pinning when traders hedge a long call position. The intuition behind the pinning is clear. The term $\frac{\partial \Delta}{\partial t}$ in eq.(4) is given by

$$\frac{\partial \Delta}{\partial t} = -\frac{n(h_1)}{\sigma\sqrt{\tau}}\frac{\log y - a\tau}{2\tau}, \tag{5}$$

where $\tau = T - t$, $a = r + \sigma^2/2$, $y = S(t)/K$ and

$$h_1 = \frac{\log y + a\tau}{\sigma\sqrt{\tau}}.$$

In Fig.(1), eq.(5) is positive for $y < e^{-a\tau}$ and negative otherways. This suggests that when the price is above the strike, $d\Delta$ is negative, inducing the hedger to sell and, doing so, he pushes the price towards the strike. Similarly when the price is below the strike, $d\Delta$ is positive inducing the hedger to buy and, doing so, he again pushes the price towards the strike. This argument then suggests that pinning is possible when the trader hedges a long position. Hedging a short position would have the opposite effect thus pushing the stock price away from the strike.

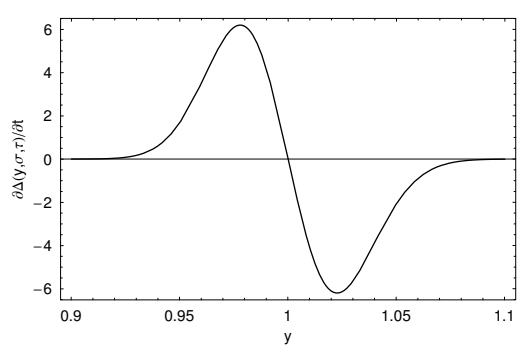

Figure 1: Time decay in function of $y = S/K$ for $\tau = 5$ days, $\sigma = 0.16$ and $a = 0$.

Frey and Stremme instead consider all the terms in the delta expansion in eq.(3) and obtain,

$$dS(t) = n\hat{L}S(t)\frac{\partial\Delta}{\partial S}dS(t) + n\hat{L}S(t)\left(\frac{\partial\Delta}{\partial t}dt + \frac{1}{2}\frac{\partial^2\Delta}{\partial S^2}d\langle S(t)\rangle\right) + \sigma S(t)dW(t),$$

or equivalently

$$\left(1 - n\hat{L}S(t)\frac{\partial\Delta}{\partial S}\right)dS(t) = n\hat{L}S(t)\left(\frac{\partial\Delta}{\partial t} + \frac{1}{2}\frac{\partial^2\Delta}{\partial S^2}\sigma^2 S^2(t)\right)dt + \sigma S(t)dW(t).$$

Rearranging we find

$$dS(t) = b(t, S(t))S(t)dt + v(t, S(t))S(t)dW(t) \tag{6}$$

with

$$b(t, S(t)) = \frac{n\hat{L}}{1 - n\hat{L}S(t)\frac{\partial\Delta}{\partial S}}\left\{\frac{\partial\Delta}{\partial t} + \frac{1}{2}\frac{\partial^2\Delta}{\partial S^2}\frac{\sigma^2 S^2(t)}{[1 - n\hat{L}S(t)\frac{\partial\Delta}{\partial S}]^2}\right\},$$

and

$$v(t, S(t)) = \frac{\sigma}{1 - n\hat{L}S(t)\frac{\partial\Delta}{\partial S}}.$$

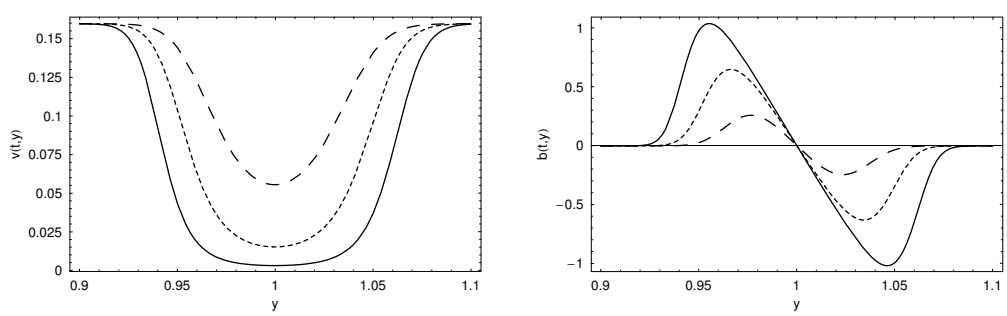

Figure 2: Drift term (right) $b(t, y)$ and volatility term (left) $v(t, y)$ in the Frey and Stremme model, as a function of $y = S/K$. The time to maturity is $\tau = 5$ days. The three lines correspond to three price elasticity: $n\hat{L} = 2.5$(solid), $n\hat{L} = 0.5$(dot), $n\hat{L} = 0.1$(dash).

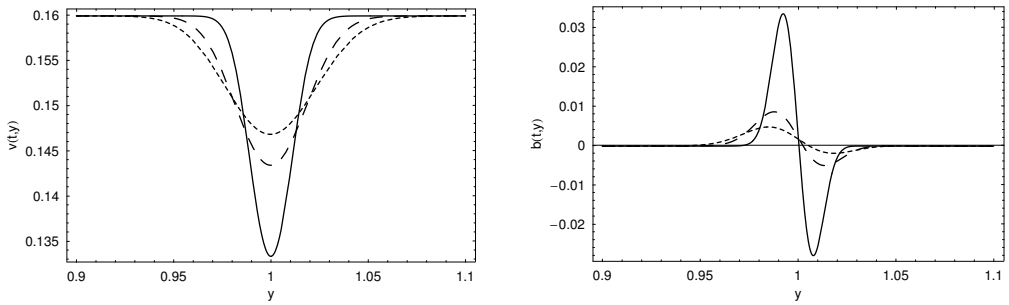

Figure 3: Drift term (right) $b(t, y)$ and volatility term (left) $v(t, y)$ in the Frey and Stremme model as a function of $y = S/K$. The price elasticity is $n\hat{L} = 0.05$. The three lines correspond to three different time to maturities: 1 day before maturity(solid), 3 days before maturity(dash), 4 days before maturity(dot).

Hence the dynamic hedging strategy generates not only a change in the drift term, as suggested by Avellaneda and Lipkin, but also a change in the volatility term (which is constant in the Avellaneda and Lipkin model).

Furthermore, the drift term $b(t, S(t))$ incorporates both the Delta time decay and the Delta convexity. This drift term is plotted in Fig.(2-right) as a function of y for different values of $\hat{L}$. The denominator in the rhs of the equation for $b(t, S(t))$ is always positive since $\frac{\partial \Delta}{\partial S} < 0$ for a long call position. For $\hat{L}$ sufficiently large, or sufficiently close to maturity, the drift term changes sign around $y = 1$ and generates pinning. When $\hat{L} \to 0$, $v(t, S(t)) \to \sigma$ and

$$b(t, S(t)) \to \frac{\partial \Delta}{\partial t} + \frac{1}{2}\frac{\partial^2 \Delta}{\partial S^2}\sigma^2 S^2(t) = \frac{n(d_1)\sigma}{\sqrt{T-t}} > 0$$

in which case pinning does not arise. The volatility term reaches its minimum value for $y = 1$, i.e. when S is close to the strike, as shown in Fig.(2-left). As $\hat{L}$ increases, the volatility term v decreases further reducing random deviation from the strike. Furthermore as we approach the maturity date the drift term becomes stronger close to strike, while the volatility (Fig.(3-left)) becomes smaller. The combined effect of these terms is not only to drive the price towards the strike but also to keep it closer to the strike as it approaches it.

We solve the Frey and Stremme model numerically. Given eq.(6), the probability density function $p(t, y)$ of being at y at time t satisfies the forward Kolmogorov equation:

$$\frac{\partial p(t,y)}{\partial t} = \frac{1}{2}\frac{\partial^2 p(t,y)}{\partial y^2} v(t,y)^2 - \frac{\partial p(t,y)}{\partial y} b(t,y) \tag{7}$$

with initial condition the delta function $\delta(\tau_0, y_0) = 1$. Eq.(7) can be solved using an implicit scheme with a adjusting mesh as we approach maturity due to the

singularity at $y = 1$ and $\tau = 0$.

Fig.(4) shows the solution of the Kolmogorov equation with initial conditions $\tau_0 = 5$ days before maturity and $y_0 = 1.04$. On the left we plot the solution for the Frey and Stremme Model and on the right the solution for the Avellaneda and Lipkin model. We see that in both cases the solution becomes bimodal as nL increases with a pronounced pick at $y = 1$. Hence both model can explain pinning as driven by hedging long call positions but the effect is stronger in the Avellaneda and Lipkin model (3.5%) than in Frey and Sremme model (1.8%). Ni et al. estimate that pinning affects 2% of optionable stocks. Our choices of parameters gives in both cases comparable values with the empirical result.

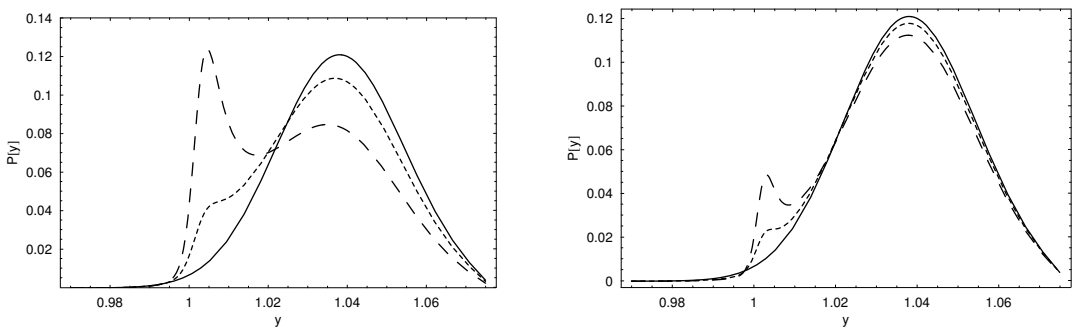

Figure 4: Solution of the Kolmogorov forward equation close to maturity, with initial condition, 5 days before maturity, $y_0 = 1.04$, and for three different hedging positions. (Left) Frey and Stremme model with $nL = 0$ (solid), $nL = 0.002$ (small dash), $nL = 0.003$ (large dash). (Right) Avellaneda model with $nL = 0$ (solid), $nL = 0.0005$ (small dash), $nL = 0.002$ (large dash).

The main assumptions behind the Frey and Stremme model (as well as of the Avellaneda and Lipkin model) is that price are lognormal, rebalancing is continuous, the price impact is linear via a constant price elesticity L, are arbitrarly large option positins can always be rehedged (demand and supply always match). Furthermore the feedback mechanism in place in these models extrapolates from actual markets condition like the order flow arrival rate and the order book shape. Empirical studies on the NYSE (see for example Lillo et al. (2003)) have shown nonetheless that the price impact function is always concave, typically well approximated by a function $dp(\omega) \sim \omega^{\alpha}$ where dp is the price change caused by an order of volume ω. The exponent α varies from $\alpha \sim 0.5$ to $\alpha \sim 0.2$ depending on stock capitalization. In the next session we introduce a microstructure model, previously studied by Daniel et al. (2003), that allows us to estimate the importance of pinning under more realistic market conditions.

3 A Microstructure Model

The aim of this section is to introduce a microstructure limit order model where a hedger rebalances his position at discrete times. The model allows us to study the impact of the hedging strategy on the order book and on the stock price and thus gives a more realistic way of determining the probability of pinning. The model implemented here has been previously introduced in Daniel et al. (2003). Alternative microstructure models have been proposed for example by Chiarella and Iori (2002), Challet and Stinchcombe (2001), Bouchaud et al (2002). The model assumes a simple random order placement of orders. All the order flows are modeled as Poisson processes. We assume that market orders arrive at a rate of ν shares per unit time, with an equal probability for buy and sell orders. Similarly, limit orders arrive at a rate of α shares per unit price and per unit time. Bids and offers are placed with uniform probability at integer multiples of the tick size Δp on a window sufficiently large around the midpoint. The midpoint is defined as $(a(t) + b(t))/2$, where $a(t)$ and $b(t)$ are respectively the best ask and the best bid at time t. When a market order arrives it causes a transaction; a buy market order removes limit orders on the offer side, and a sell market order removes limit orders on the bid side. Limit orders can also be removed spontaneously by being canceled or by expiring which, in the model, happens at a constant rate of δ per unit time. The size of the limit and market orders are sampled from a log normal distribution with mean and variance one. In Daniel et al. (2003) it is shown that two parameters characterize the shape of the book: the asymptotic depth α/δ and $\epsilon = 2\delta/\nu$. The asymptotic depth gives the number of shares per price interval far from the midpoint; ϵ determines the depth at the bid and at the ask. ϵ also determines the price impact function which is linear for $\epsilon > 0.1$ and concave for smaller values of ϵ. We calibrate the model by assuming that market orders arrive with a frequency of about two a minute, such as $\nu = 0.16$ and $\delta t = 0.08$ minutes. The other parameters are initially set to $\alpha = 0.31$, $\delta = 0.08$, which gives a value of $\epsilon = 1$. We will later compare it with $\epsilon = 0.025$. For the price tick we choose $\Delta p = 0.02$.

To study pinning we focus on the impact of a trader that rehedges a long call position four times a day. The option expires in five days. In the model, hedging is achieved by submitting market orders and not limit orders because immediacy in execution is important for the hedger. When the stock price increases, the hedger places a market order to sell the variation of $\Delta(S, t)$, and when the stock price decreases, the trader places a market order to buy the variation of $\Delta(S, t)$. We first compare the shape of the stock's drift and volatility in the simulated process with the theoretical models proposed in the previous section.

In Fig.(5), we show that pinning still arises in the microstructure model. As expected, the pinning effect is stronger when the hedger position is bigger. In the model the maximum amount of orders that the hedger can submit is limited by the overall trading activity in the market. In fact if the amount of orders submitted by the hedger is higher than the amount of orders submitted by the rest of the market, the book becomes empty. Hence there is a maximum option position that can be simulated. Thus there is a limit to the pinning generated by our model. Nonetheless the pinning probabilities we obtain in Fig.(5) are comparable with the empirical findings. Fig.(5) also shows that pinning is also more likely to arise when ϵ is large, i.e. when the price impact is linear. This provides a justification for this assumption in the theoretical models.

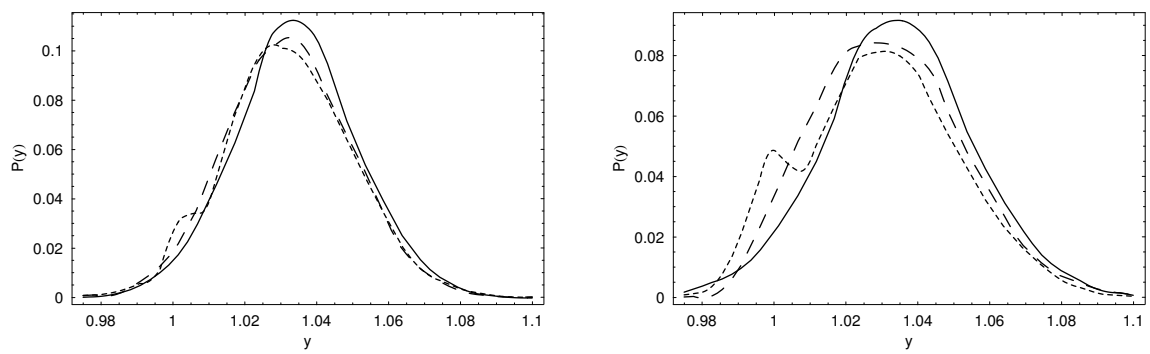

Figure 5: Stock price distribution without the hedger (solid) and with the hedger $n = 600$ (dash) and $n = 1000$ (dot) for two different ϵ: $\epsilon = 0.025$ (left), $\epsilon = 1.$ (right) under microstructure model

We then test if the mechanisms that generate pinning are consistent with the theoriticaI model. To this purpose, we study the behaviour of the drift and the volatility in the microstructure model. Given that an instantaneous drift $b(t, S)$ and volatility $v(t, S)$ cannot be defined here, we measure the average drift and volatility over a period $\Delta t = t_1 - t_0$ and averaged them over a number of trajectories M, i.e.

$$\mu(t_0) = \frac{1}{M} \sum_{j=1}^{M} \mu_j, \tag{8}$$

where m is the number of steps between t_0 and t_1, $\mu_j = \frac{1}{m} \log \frac{S^j(t_0+\Delta t)}{S^j(t_0)}$, and

$$\sigma(t_0) = \frac{1}{M} \sum_{j=1}^{M} \sqrt{\frac{1}{m-1} \sum_{i=1}^{m} (r(t_i) - \mu_j)^2}, \tag{9}$$

with $r(t_i) = \log(S(t_i + \Delta t/n)/S(t_i))$. We have chosen here to simulate $M = 10000$ paths, with $\Delta t = 2$ days, to allow the effect of hedging to be sufficiently incorporated in the stock price. We take the time to maturity$\tau = 5$ days and repeat

the simulation starting from initial condition S_0. In Fig.(6), we observe that the microstructure model generates a similar drift profile as in the theoretical models but the volatility differs. Here the volatility increases as we approach the strike, while in Frey & Stremme it decreases as we approach the strike.[2]

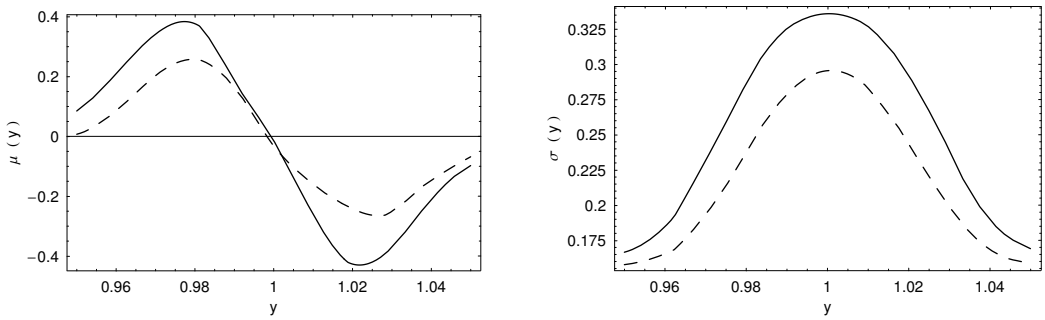

Figure 6: Drift and volatility under microstructure model for 2 differents ϵ: $\epsilon = 0.025$ (dash), $\epsilon = 1$ (solid) for $n = 1000$.

4 Conclusion

In this paper, we compare three different models that are all capable of generating pinning by hedging long call positions. The three models share a common pinning mechanism, i.e. an excess drift generated by delta hedging that pushes the stock price towards the strike. The model differs in the way hedging impacts on the volatility which remains constant in the Avellaneda & Lipkin model, present a minimum around the strike in the Frey & Stremme model and a maximum in the microstructure model. Empirical analysis is required to clarify which model generates the most realistic behaviour.

[2]We check that, using the same method, we recover the correct shape for the drift and the volatility when simulating the SDE in eq. 5 via Monte Carlo. We also checked that without the hedger, the drift $\mu(t, S)$ and the volatility $\sigma(t, S)$ stay constant.

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