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Allocating Security Expenditures under Knightian Uncertainty: an Info-Gap Approach*

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Abstract

We apply the information gap approach to resource allocation under Knightian (non-probabilistic) uncertainty in order to study how best to allocate public resources between competing defense measures. We demonstrate that when determining the level and composition of defense spending in an environment of extreme uncertainty *vis-a-vis* the likelihood of armed conflict and its outcomes, robust-satisficing expected utility will usually be preferable to expected utility maximization. Moreover, our analysis suggests that in environments with unreliable information about threats to national security and their consequences, a desire for robustness to model misspecification in the decision making process will imply greater expenditure on certain types of defense measures at the expense of others. Our results also provide a positivist explanation of how governments seem to allocate security expenditures in practice.

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"*Reports that say that something hasn't happened are always interesting to me, because as we know, there are known knowns; there are things we know we know. We also know there are known unknowns; that is to say we know there are some things we do not know. But there are also unknown unknowns – the ones we don't know we don't know.*"

Donald Rumsfeld

U.S. Secretary of Defense

1 Introduction

When allocating resources, policy makers, like households, face trade-offs. Many of the trade-offs households encounter can reasonably be modeled in a deterministic setting—their incomes may be uncertain, but when deciding between a new refrigerator or a new television set, we generally assume that members of a household know the marginal utility they will derive from each. By contrast, policy makers nearly always confront decisions in which the connection between allocations, and the desirability of outcomes, are highly uncertain. No one can predict precisely by how much a dollar transferred between different components of the public health budget will affect an individual citizen's longevity. Nonetheless, policy makers know a great deal about the prevalence of infectious diseases, heart problems, and cancer in the population as a whole, and the efficacy of different treatments for large samples of patients. Given this knowledge, decision makers are able to derive fairly reliable probabilistic models that link different allocations of funding with the moments of a distribution of outcomes.

Unlike public health decisions, allocating resources for national security involves decisions where experiments are not possible (or at least unwise), and previous experience provides little or no useful data. A policy maker whose country is threatened by a hostile neighbor may know little about the probability distribution of possible damage or losses their country could suffer in the event of armed conflict, or how they relate to choices between alternative allocations of resources to different security and defense measures. Indeed, the probability that an adversary will launch an attack may itself be unknown, but nonetheless dependent on the type of defense capabilities our policy maker has chosen. Conditioned on an attack actually taking place, the damages and losses the attack will cause have a distribution that is only vaguely known. Moreover, this distribution may itself depend in a complex way on the quality of intelligence, civil defense preparedness, and the structure, size and deployment of military forces. Similar tradeoffs exist in allocating resources between homeland security measures and armed forces, although the nature of the risks and their relationships to defense measures is obviously different.

When allocating resources to enhance defense and security, policy makers have to decide both

how much to devote to defense spending, and how best to distribute that spending between a wide range of competing options, without knowing the effects of their decisions on the ensuing distribution of possible damages—or at best having only some *qualitative* knowledge about these relationships. Experience simply does not supply enough information to accurately derive the true probability distribution of all these different kinds of risks. Nor does it deliver a clear picture of the relationships between different levels, and types of security expenditures, a nation may deploy to protect itself from war and these risks probabilities and distributions. Indeed, past history provides little guidance for predicting the losses in human life and economic disruption that future conflicts may entail.

In this paper we demonstrate how information gap theory, a methodology designed to aid in decision making under Knightian (non-probabilistic) uncertainty, can be usefully applied to the question of how best to allocate resources for national defense. We focus on the example of defense expenditures, because defense policy typically involves making hard choices, with limited and often unreliable information, in an environment where limited resources do not permit the unconstrained funding of all the possible measures that are available. By implementing the info-gap methodology, policy makers combine what they know about security threats, and their relationships to installed military capabilities, with how reliable or unreliable they suspect that information may be. *By distinguishing between these two attributes of the random nature of threats to national security, and the effectiveness of possible countermeasures, we can separate the implications of the underlying threats from the issue of how uncertain we are about them.*

We demonstrate that when choosing the level and type of defense measures that affect security in different ways, when probablistic information about threats is unreliable, it is better to robust-satisfice the citizen's expected utility rather than to attempt to maximize it.[1] Moreover, our analysis highlights one rationale for heightened spending on some types of defense measures when there is little reliable information about the random nature of threats to national security, thus providing a possible positivist explanation for the way governments are allocating these expenditures today, and how they have done so in the past.

In section 2 we describe the general formulation of the info-gap methodology. In section 3 we consider the dilemma faced by a hypothetical policy maker who must choose how much to spend on each of two different types of military technology. The first type of expenditure involves investment in large units employing traditional industrial-age military technology. This, the more conservative approach, provides relatively high insurance against severe and widespread losses. The second type of expenditure is built around the 'Revolution in Military Affairs'

[1] In this paper satisficing implies an allocation of defense resources which maximizes expected utility for a satisfactory level of robustness to model misspecification.

(RMA) doctrine, and involves a heavier reliance on precision munitions, electronic warfare, and intelligence, as well as small, highly mobile and specially trained military units.[2]

Reliance on forces built to operate according to the RMA doctrine can under the most optimal conditions limit the severity of casualties and other losses, but is more prone to catastrophic failure, particularly in the event of a massive invasion by a large military force. As defense expenditures are more heavily weighted towards this RMA type of expenditure at the expense of traditional weapons systems, expected losses conditioned on suffering an attack decline, but there is a possibly greater risk of extremely high damage. We assume throughout that the risk of being attacked and the probability distribution of damage conditioned on an attack occurring, are functions of both the size and structure of the military force policy makers have chosen.

We demonstrate that when policy makers must operate in an environment in which information about the relationship between threats and different allocations for defense is unreliable, expected utility maximization may not provide the best guidance for making decisions, because it is completely non-robust to model misspecification. Using a hypothetical specification of security risks and the cost effectiveness of alternative defense measures, we show that the desire of policy makers to attain some degree of robustness to model misspecification, will have profound effects on both the level of resources devoted to defense, and the distribution of these expenditures across the two possible types of defense measures.

2 Formulation

Consider a country facing various threats to its national security. These threats may take various forms, including invasion by an aggressive neighbor or a terrorist attack. Policy makers perceive threats to security from all sources as a bivariate distribution that includes both the event of being attacked, and the damage the country will sustain conditional on the attack taking place. In contrast to standard stochastic optimization problems, this distribution itself is uncertain. Policy makers must decide what portion of the economy's resources they will devote to countering these risks, as well as how to allocate this expenditure between different defense measures, where each measure has a different effect on both the risk and the potential damage from an attack. This defense resource allocation dilemma is embedded within a standard economic framework in which the representative risk-averse individual in this country derives utility $u(c)$ only from

[2]First elaborated during the 1980's by Soviet military theorists, in particular Marshal Nikolai Ogarkov, then chief of the Soviet General Staff, the RMA doctrine of war has already had a profound effect on Western military planning. The move to adapt the U.S. military to RMA type warfare was spearheaded by the Department of Defense's Office of Net Assessment, and the man who has headed it since 1973, Andrew Marshall, and the vice chairman of the Joint Chiefs of Staff between 1994 to 1996, Admiral William Owens. See Eliot A. Cohen (1996).

consumption, c. The threats considered here, and the effect of any countermeasures, affect the resulting level of utility by their influence on the amount of resources that remains for consumption and the degree of risk they ultimately face.

Normalizing the economy's resources to 1, the policy maker must choose the fraction of all resources it will devote to each of a number of different risk-mitigating expenditures $\chi = (\chi_1, \ldots, \chi_N)$. Without government debt, we require $\sum_i^N \chi_i \leq 1$. Any government expenditure detracts from the resources available for consumption, so $c = 1 - \sum_i^N \chi_i$. On the other hand these government expenditures reduce the fractional loss ψ in resources resulting from the security risks faced by citizens in the economy, where $0 \leq \psi \leq 1$. Let $\chi_{\mathrm{c}} = 1 - \sum_i^N \chi_i$, be the fraction of GDP devoted to consumption. Denote the utility in the ideal situation where none of the risks materialize by $u_{\mathrm{c}} = u(\chi_{\mathrm{c}})$.

The probability density function (pdf) of realized threats, conditioned on risk mitigating expenditures, is $p(\psi|\chi)$—a probability distribution unknown to policy makers. The best available estimate of $p(\psi|\chi)$ is denoted $\widetilde{p}(\psi|\chi)$ but it is incontrovertible that $\widetilde{p}(\psi|\chi)$ is highly unreliable. We assume that the probability that a representative agent will suffer a loss in his welfare from an attack is P_{w}. The value of P_{w} is highly uncertain and its best estimate, $\widetilde{P}_{\mathrm{w}}$, depends on both the level and distribution of defense expenditures.

Let $R(\chi|p, P_{\mathrm{w}})$ be the expected utility resulting from defense expenditure χ, when the probability of the threat being realized is P_{w}, and the pdf of the damage ψ is $p(\psi|\chi)$:

$$R(\chi|p, P_{\mathrm{w}}) \quad = \quad \left(\int_0^1 u\left[(1-\psi)\chi_{\mathrm{c}}\right] p(\psi|\chi)\, \mathrm{d}\psi \right) P_{\mathrm{w}} + (1 - P_{\mathrm{w}}) u_{\mathrm{c}} \tag{1}$$

Higher expected utility is preferable over lower expected utility, but R_{c} is the lowest acceptable level of expected utility. It is a reward aspiration or a 'reservation reward'.

The info-gap model is a family of nested sets of probability models $p(\psi|\chi)$ and $P_{\mathrm{w}}(\chi)$, indexed by α, which represents the degree of uncertainty in the policy maker's best estimate of the chances of any damage occurring and the conditional distribution of its level. We denote this info-gap model by $\mathcal{F}[\alpha, \widetilde{p}(\psi|\chi), \widetilde{P}_{\mathrm{w}}(\chi)]$, where $\alpha \geq 0$.

Info-gap models obey two axioms:

(i) *Nesting* asserts that the range of possible pdfs increases as α increases:

$$\alpha < \alpha' \quad \Longrightarrow \quad \mathcal{F}[\alpha, \widetilde{p}, \widetilde{P}_{\mathrm{w}}] \subset \mathcal{F}[\alpha', \widetilde{p}, \widetilde{P}_{\mathrm{w}}] \tag{2}$$

(ii) *Contraction* asserts that when $\alpha = 0$, the estimated models are the only possibilities:

$$\mathcal{F}[0, \widetilde{p}, \widetilde{P}_{\mathrm{w}}] = \left\{ \widetilde{p}, \widetilde{P}_{\mathrm{w}} \right\} \tag{3}$$

These two axioms endow α with its meaning of a *horizon of uncertainty.*

Let $\mu(\chi, \alpha)$ be the lowest expected reward, given defense expenditures χ, over an info-gap model α around $\widetilde{p}$ and $\widetilde{P}_{\mathrm{w}}$. That is:

$$\mu(\alpha, \chi) = \min_{(p, P_{\mathrm{W}}) \in \mathcal{F}(\alpha, \widetilde{p}, \widetilde{P}_{\mathrm{w}})} R(\chi | p, P_{\mathrm{w}}) \tag{4}$$

The value $\widehat{\alpha}(\chi, R_{\mathrm{c}})$ is the robustness (to uncertainty in $\widetilde{p}$ and $\widetilde{P}_{\mathrm{w}}$) of security expenditures χ with reward-aspiration R_{c}. It is the greatest range of Knightian uncertainty, α, up to which all probability models in $\mathcal{F}$ result in reward no less than R_{c}, given expenditure χ:

$$\widehat{\alpha}(\chi, R_{\mathrm{c}}) = \max \{\alpha : \mu(\alpha, \chi) \geq R_{\mathrm{c}}\} \tag{5}$$

The robustness function displays a fundamental trade-off between reward and robustness to uncertainty: robustness decreases as the aspired reward increases (Ben-Haim, 2006):

$$R_{\mathrm{c}} > R'_{\mathrm{c}} \quad \Longrightarrow \quad \widehat{\alpha}(\chi, R_{\mathrm{c}}) \leq \widehat{\alpha}(\chi, R'_{\mathrm{c}}) \tag{6}$$

Furthermore, if the aspiration is for the greatest reward expected with the estimated distribution, then the robustness is zero (Ben-Haim, 2005):

$$\chi^* = \arg\max_{\chi} R(\chi | \widetilde{p}, \widetilde{P}_{\mathrm{w}}), \quad R_c = R(\chi^* | \widetilde{p}, \widetilde{P}_{\mathrm{w}}) \quad \Longrightarrow \quad \widehat{\alpha}(\chi^*, R_{\mathrm{c}}) = 0 \tag{7}$$

This means that if the estimated models $\widetilde{p}(\psi|\chi)$ and $\widetilde{P}_{\mathrm{w}}$ are used to choose an expected-utility-maximizing allocation χ^*, then this aspiration has zero robustness to uncertainty in these models.

Since more robustness is preferable to less, at the same level of satisficed utility, the decision maker may wish to choose χ to satisfice the utility and to maximize the robustness. This is an **info-gap robust-satisficing** decision approach, which is formally defined:

$$\widehat{\chi}(R_{\mathrm{c}}) = \arg\max_{\chi} \widehat{\alpha}(\chi, R_{\mathrm{c}}) \tag{8}$$

In this paper the policy maker chooses the value of χ that maximizes the lowest possible value of expected utility over a set of probabilities and distribution functions indexed by α around the best estimates. For any desired level of robustness α, the decision maker will choose χ to maximize $\mu(\alpha, \chi)$ as defined in (4). Note that for any aspiration level R_c, $\mu[\widehat{\chi}(R_c), \widehat{\alpha}(\widehat{\chi}(R_c), R_c)] \equiv R_c$.

3 An Illustration with Two Types of Security Expenditure

3.1 Background

How do we quantify security threats and the losses they may generate, and what are the different possible security expenditures that are meant to counter them? Consider first a threat from

Conflict	Total Direct Costs		People Mobilized		Fatalities	
	Millions of 2002$	*Percent of GDP*	*Thousands*	*Percent of Pop.*	*Numbers*	*Percent of Pop.*
Revolutionary War (1775 – 1783)	2.2	63%	200	5.70%	4,435	0.127%
War of 1812 (1812 – 1815)	1.1	13%	286	3.80%	2,260	0.030%
Mexican War (1846 – 1848)	1.6	3%	79	0.40%	1,733	0.008%
Civil War (1861 – 1865)	62	104%	3,868	11.10%	184,594	0.538%
Span. Amer. War (1898)	9.6	3%	307	0.40%	385	0.001%
World War I (1917 – 1918)	190.6	24%	4,744	4.60%	53,513	0.052%
World War II (1941 – 1945)	2,896.3	130%	16,354	12.20%	292,131	0.219%
Korea (1950 – 1953)	335.9	15%	5,764	3.80%	33,651	0.022%
Vietnam (1964 – 1972)	494.3	12%	8,744	4.30%	47,369	0.023%
First Gulf War (1990 – 1991)	76.1	1%	2,750	1.10%	148	0.000%

Table 1: American Costs and Casualties from Major Wars. *Source*: William D. Nordhaus, "The Economic Consequences of a War with Iraq" in *War with Iraq*, ed. Kaysen, *et. al.*

a conventional adversary. Table 1 presents the direct expenses of major U.S. wars along with U.S. fatalities. In proportion to the size of the economy, the heaviest direct economic costs were incurred during World War II; however, in terms of loss of life relative to the size of the population, the Civil War was far deadlier. Obviously the latter conflict, fought entirely on American soil, generated the greatest loss of capital, damage to infrastructure and disruption of output. In the aftermath of World War I, Bogart (1919) began developing the tools to measure, compare, and aggregate all these different costs of war, by including not only the direct costs in military expenditure as well as physical destruction, but the indirect costs associated with the capitalized values of losses in life and lost production. According to his calculations, the share of direct costs incurred by all the combatants of World War I amounted to only 55% of the total losses the war generated.

Broadberry and Howlett (1998) calculate that the UK spent approximately half of its GDP fighting World War II during the years 1940 to 1944. In addition it suffered losses of physical

capital that amounted to 89% of GDP in 1938 (see Mitchell 1980), and human capital losses (calculated conservatively in terms of just the schooling invested in those killed) of 2.5% of GDP in 1938. By any measure Soviet losses were far higher. During 1942 and 1943 defense expenditures in the Soviet Union reached 61% of GDP, losses of physical capital amounted to 223% of pre-war GDP and losses of human capital were 109% (Harrison (1998)). Of course these figures do not include the extraordinary privations suffered by those living under German occupation during much of this period. To study the cost of World War II for the United States, Rockoff (1998) employs a counterfactual approach developed by Goldin and Lewis (1975) to study the Civil War. According to his estimates the total present value of foregone consumption that can be attributed to both direct and indirect losses generated by the war equals 2.27 years of consumption in 1941.

As most of its effects are indirect, the impact of terror is not as well understood. Abadie and Gardeazabal (2003) estimate that terrorism in the Basque country of Northern Spain has reduced GDP by 10%. Similarly, recent estimates of the loss in GDP that can be attributed to three years of recurring terrorism against Israel is also approximately 10% (Eckstein and Tsiddon (2004) and Persitz (2005)). Bram, Orr and Rapaport (2002) estimate the total cost of one incident, the September 11, 2001, attack on the World Trade Center in New York, including lost lifetime earnings of those killed, to be between $33 billion and $36 billion. What is clear is that large-scale conventional warfare is far more costly than any losses associated with terror—Hess (2003) calculates that for countries that have experienced conflict between 1960-1992 (nearly all of it civil war or terrorism in this period), the loss in welfare associated with these conflicts is on average equivalent to a permanent 8% drop in their consumption.

Fragmentary anecdotal and quantitative evidence exists concerning the impact of military expenditures on the probability distribution of war-related damage. Rohlfs (2005) estimates that the marginal effectiveness of a U.S. tank in Western Europe for 164 battles during World War II, was twenty-four times the effectiveness of a single infantryman but eighty-seven times as expensive to use. The discrepancy is explained by the higher casualties associated with intensive use of infantry, implying that the U.S. government assigned a value of approximately one million dollars (in 2003 dollars) to each soldier's life saved on the battlefield. Although Rohlfs' study involves calculating ex-post a relatively simple trade-off in a single theater of a war in its fifth and sixth year (1944-1945), his estimates contain relatively large standard errors and vary across different sub-samples. By contrast, policy makers must determine ex-ante both the overall effectiveness of defense expenditures and their optimal allocations, at a stage when the nature, scale, and even eventuality of a conflict may be only hypothetical. In the years prior to being attacked in 1941, the U.S.S.R. engaged in a massive rearmament program that,

according to Bergson (1961), lowered per-capita consumption between 1937 and 1940 by as much as 8.4%. However, not only did rearmament fail to deter Hitler's invasion as the Soviets had hoped, but because of serious military miscalculations, much of the arms and manpower was squandered during the summer and fall of 1941 without seriously slowing the German advance (Harrison (1985)).

3.2 Basic Structure

Consider an economy in which resources are allocated between civilian consumption and two different types of security-related public expenditure. We divide security related public expenditures into two broad categories, denoted χ_1 and χ_2. Both χ_1 and χ_2 are measured as shares of GDP, and we ignore the possibility of international borrowing. We define a probability density function (pdf) $p(\psi|\chi)$ as the damage the nation sustains in terms of lost GDP in the event that it is attacked, and assume its shape is influenced by both the overall amount of resources devoted to security, $\chi_1 + \chi_2$, and also by how resources are divided between χ_1 and χ_2.

All military expenditures fall into one of two broad categories. First, there are the development and maintenance of large military formations, composed of large numbers of troops receiving traditional military training to serve as infantrymen or to operate armor, artillery, battleships and bombers. We denote this, the more traditional type of military expenditure, and the one most closely associated with twentieth century warfare, as χ_1. The second, type of expenditure incorporates recent innovations in military technology and tactics, based on the intensive use of information technology (IT) intelligence, high precision weaponry, and command and control systems. This type of expenditure, which we denote χ_2, is associated with the term 'Revolution in Military Affairs' (RMA).

The possible damages from war is represented by the probability density function over the possible fraction of output destroyed during an armed conflict, up to a maximum conceivable fraction z. However, given the high degree of uncertainty surrounding the conflict, we assume that policy makers do not know the true pdf of damage. Policy makers' best (but highly uncertain) estimate of the damage pdf, conditional on being attacked, and the chosen allocation of defense expenditure χ_1 and χ_2 are denoted by $\widetilde{p}(\psi|\chi_1, \chi_2)$. Given the long planning horizon necessary to prepare for armed conflict, the relevant unit of time in this model is a decade.

The functional form for the best estimate of the damage density function reflects all available knowledge about possible damages from threats to national security, and how these risks are affected by both types of security expenditures. In the appendix we provide the specification of the probability density function that is used in our illustration. It suffices here to note the two most salient features implied by our specification, which capture the essence of the tradeoff

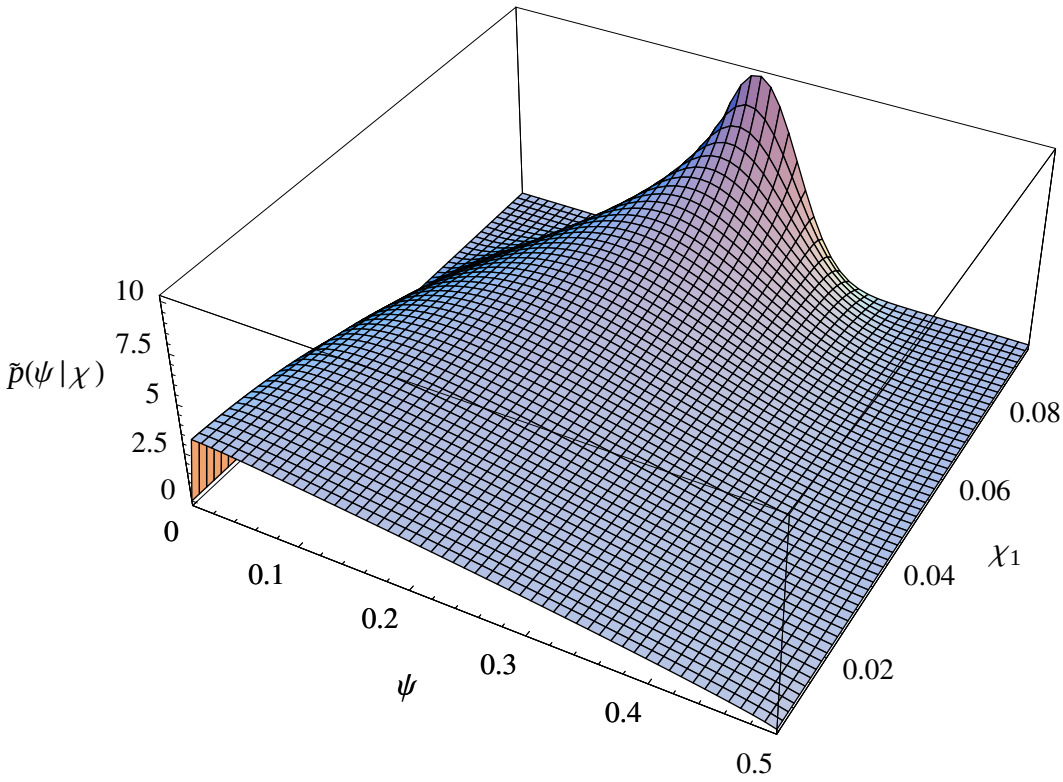

Figure 1: The best estimate of the probability density of damage, conditional on defense expenditure $\widetilde{p}(\psi|\chi)$ as in (20), for different values of χ_1 where total defense expenditure is $\chi_1 + \chi_2 = .1$, and the maximum possible damage (as a share of GDP) is $z = 1/2$.

assumed to exist between the two alternative types of defense expenditure:

1. Mean damage generally declines as total security expenditures increase. That is, we expect behavior along the lines of:
$$\frac{\partial \mathrm{E}(\psi)}{\partial(\chi_1 + \chi_2)} < 0 \tag{9}$$

2. Mean damage and extreme damage respond in opposite directions to increases in RMA-type expenditure, holding total security expenditure fixed. That is, we expect to generally observe:
$$\left(\frac{\partial \mathrm{E}(\psi)}{\partial \chi_2}\bigg|_{\chi_1+\chi_2} \right) \times \left(\frac{\partial \mathrm{Prob}(\psi > 0.4z)}{\partial \chi_2}\bigg|_{\chi_1+\chi_2} \right) < 0 \tag{10}$$

In Figure 1 we graphically present the density function $\widetilde{p}(\psi|\chi)$, based on the specification in the appendix, where the maximum damage $z = 1/2$, holding total defense expenditures $\chi_1 + \chi_2$ constant at ten percent of GDP, and varying the value of the traditional military expenditure χ_1 between the values 0 and 0.1.

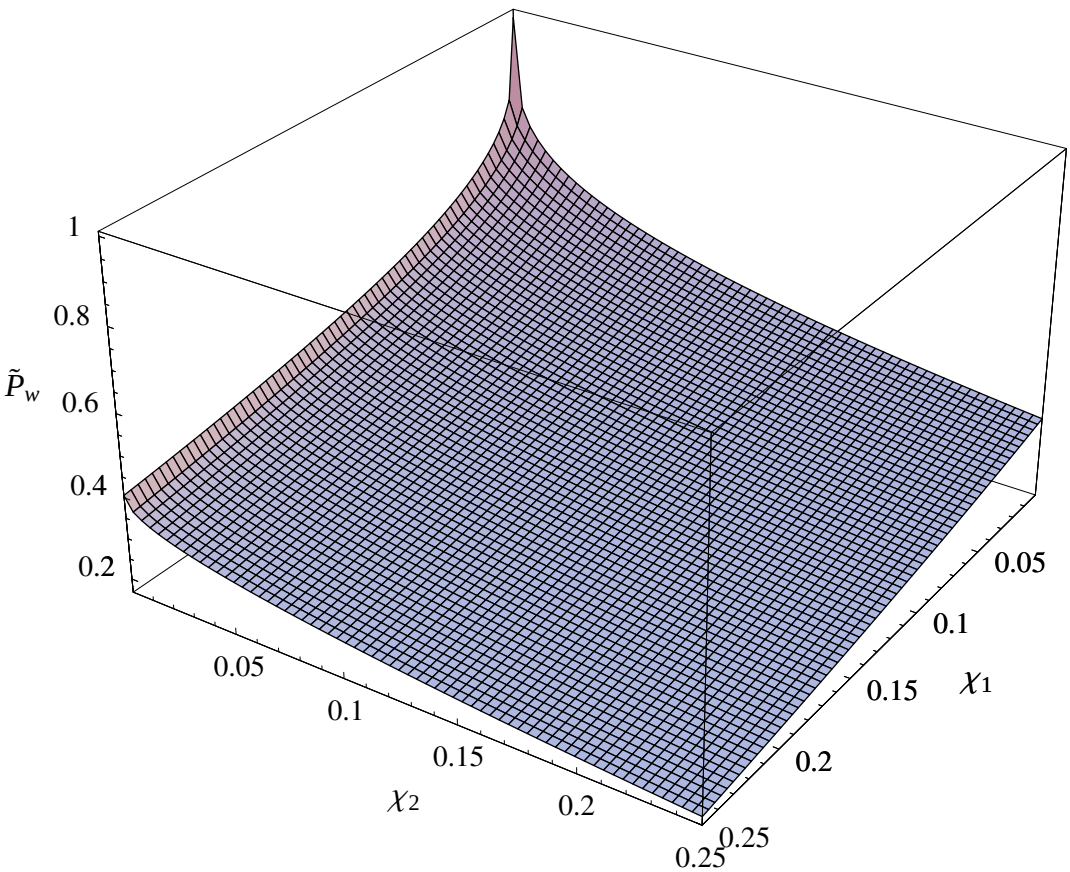

Figure 2: The best estimate of the probability of being attacked $\widetilde{P}_{\mathrm{w}}(\chi)$ as in (24), for different values of χ_1 and χ_2, where $z = 1/2$.

The probability of suffering an attack over the course of a decade is also subject to uncertainty, and is also a function of the overall size as well as distribution of defense expenditures. We denote the probability of attack by $P_{\mathrm{w}}(\chi)$ and the best (but highly uncertain) estimate of this function is $\widetilde{P}_{\mathrm{w}}(\chi)$.[3] The salient feature, which we illustrate in Figure 2, is that $\widetilde{P}_{\mathrm{w}}(\chi)$ is a decreasing function of both types of expenditure.

3.3 Info-Gap Model of Uncertainty

The density function $\widetilde{p}(\psi|\chi)$ is the best estimate of the pdf of damage of an attack, given security allocations χ.[4] However, this estimate is based on fragmentary and unreliable evidence, and hence contains potentially serious but unidentifiable errors. The same is true for the estimated probability of attack, $\widetilde{P}_{\mathrm{w}}(\chi)$. The true values deviate from these estimates by unknown amounts.

We use a fractional error info-gap model to represent the info-gaps in both the pdf of the

[3]See (24) in the appendix for precise function used in this example.

[4]See (20) in the appendix for precise function used in this example.

damage and the probability of attack (Ben-Haim, 2006). Let $\mathcal{P}$ denote the set of all pdfs on [0, 1]. Our info-gap model is the unbounded family of sets of pdfs $p(\psi)$ and probabilities P_{w}:

$$\mathcal{F}(\alpha, \widetilde{p}, \widetilde{P}_{\mathrm{w}}) = \Big\{ p(\psi),\, P_{\mathrm{w}} : \quad p(\psi) \in \mathcal{P},\ |p(\psi) - \widetilde{p}(\psi|\chi)| \le \alpha \widetilde{p}(\psi|\chi),\ \text{for all } \psi$$
$$0 \le P_{\mathrm{w}} \le 1,\ |P_{\mathrm{w}} - \widetilde{P}_{\mathrm{w}}(\chi)| \le \alpha \widetilde{P}_{\mathrm{w}}(\chi) \Big\}, \quad \alpha \ge 0 \tag{11}$$

The family of sets $\mathcal{F}(\alpha, \widetilde{p}, \widetilde{P}_{\mathrm{w}})$ contains every pdf $p(\psi)$ that deviates proportionally from the estimated density $\widetilde{p}(\psi|\chi)$, at any level of damage ψ, by no more than α, and also all the probabilities of being attacked P_{w} which differ proportionally from the estimated value $\widetilde{P}_{\mathrm{w}}(\chi)$, also by no more than α. The value of the fractional error α is unknown, so the info-gap model is not a single set, but rather an unbounded family of nested sets of possible pdfs and probabilities. Since α is unbounded, there is no worst case.

3.4 Robustness Function

The expected utility if an attack occurs, based on the estimated pdf of damage, is:

$$\widetilde{r}(\chi) = \int_0^1 u\left[(1-\psi)\chi_{\mathrm{c}}\right] \widetilde{p}(\psi|\chi)\, \mathrm{d}\psi \tag{12}$$

Recall that the utility if an attack does not occur is u_{c}, equal to $u(\chi_{\mathrm{c}})$. We assume that this is greater than the estimated expected utility in the case of attack:

$$\widetilde{r}(\chi) < u_{\mathrm{c}} \tag{13}$$

The total estimated expected utility, based on the estimated pdf of damage $\widetilde{p}(\psi|\chi)$ and the estimated probability of attack $\widetilde{P}_{\mathrm{w}}(\chi)$, is:

$$\widetilde{R}(\chi) = \widetilde{P}_{\mathrm{w}}(\chi)\widetilde{r} + (1 - \widetilde{P}_{\mathrm{w}}(\chi))u_{\mathrm{c}} \tag{14}$$

The expected utility for arbitrary $p(\psi|\chi)$ and P_{w} is specified in (1), and the robustness function is defined in (5). We derive the robustness function, based on the info-gap model (11) and on the assumption (13), in the appendix, section 5.3. Suppressing the notational dependence on χ, the robustness function is:

$$\widehat{\alpha}(\chi, R_{\mathrm{c}}) = \begin{cases} \dfrac{(\widetilde{r} - u_{\mathrm{c}} - \delta_r)\widetilde{P}_{\mathrm{w}} + \sqrt{(\widetilde{r} - u_{\mathrm{c}} - \delta_r)^2 \widetilde{P}_{\mathrm{w}}^2 + 4\delta_r \widetilde{P}_{\mathrm{w}}(\widetilde{R} - R_{\mathrm{c}})}}{2\delta_r \widetilde{P}_{\mathrm{w}}} & \text{if } \widetilde{R} \ge R_{\mathrm{c}} \\ 0 & \text{else} \end{cases} \tag{15}$$

where:

$$\widetilde{r}_1 = \int_0^{\psi_{\mathrm{m}}} u\left[(1-\psi)\chi_{\mathrm{c}}\right] \widetilde{p}(\psi|\chi)\, \mathrm{d}\psi \tag{16}$$

$$\widetilde{r}_2 = \int_{\psi_{\mathrm{m}}}^1 u\left[(1-\psi)\chi_{\mathrm{c}}\right] \widetilde{p}(\psi|\chi)\, \mathrm{d}\psi \tag{17}$$

$$\delta_r = \widetilde{r}_1 - \widetilde{r}_2 \tag{18}$$

and ψ_{m} is the median of $\widetilde{p}(\psi|\chi)$. Given that marginal utility is positive, $\widetilde{r}_1 > \widetilde{r}_2$, and therefore $\delta_r > 0$. This, together with (13), implies that $\widetilde{r} - u_{\mathrm{c}} - \delta_r < 0$.

3.5 Numerical Results

We adopt the constant-risk-aversion specification for the utility function $u(c) = c^{1-\gamma}/(1-\gamma)$, where $\gamma > 0$. The best estimates of the pdf of damage is defined in (20)–(22) in the Appendix, and the probability of suffering an attack is defined in (24). The value of $\widetilde{R}$ in (14) is expressed in terms of utility. In our economy the representative individual has a maximum of a single unit of consumption, from which defense expenditures and any damage to the economy are deducted. Rather than report our results in units of utility, we convert the values of $\widetilde{R}$ in (14) into their consumption equivalents.[5]

Figure 3 contains two examples corresponding to maximum potential damages $z = 1/2$ and $z = 1$, for different values of χ_1 and χ_2 in the grid $[0, 0.4] \times [0, 0.4]$. The frames are shown with levels of robustness equal to 0, 1, 2 and 3, corresponding to immunity to 0, 100%, 200% and 300% error in the probability models. For the frame with robustness $\widehat{\alpha} = n$, each contour is indexed by an R_c value, and consists of all defense expenditures (χ_1, χ_2) at which $\widehat{\alpha}(\chi, R_c) = n$.

Consider the four panels on the right-hand side of Figure 3 that correspond to the maximum possible damage of 100%, $z = 1$—an attack has the potential to drive consumption all the way to zero. Recall from (7) that a policy maker who wishes to maximize the expected utility must accept zero robustness to model misspecification. Looking at the upper right-hand panel of Figure 3, we find that if policy makers maximize expected utility under $\widetilde{p}(\psi|\chi)$ and $\widetilde{P}_{\mathrm{w}}(\chi)$, the greatest level at which the expected utility can be satisficed is obtained if they choose to devote 7.4% of GDP to defense, allocating 2.7% to traditional military expenditures, χ_1, and 4.7% to expenditures on RMA, χ_2. As illustrated in Table 2, under expected utility maximization the policymakers are willing to tolerate a high probability of being attacked over the course of a decade ($\widetilde{P}_{\mathrm{w}}(\chi)$=0.517), a relatively high level of expected damage if the attack occurs ($\mathrm{E}(\psi \mid \chi)$=0.355), and a fairly large probability of suffering losses of over 40% of GDP, again if the attack occurs ($\Pr\{\psi \geq 0.4z\}$=0.392). On the other hand, if policymakers are unsure of the reliability of their probabilistic estimates and require a robustness level of 100% ($\widehat{\alpha} = 1$), the amount of GDP devoted to defense increases to 28.8% of consumption, with 18.8% of GDP devoted to traditional military expenditures, and 10% devoted to RMA. The probability of suffering an attack, the expected damage the country will sustain if an attack does occur, and the probability that any losses will exceed 40% of GDP all drop at these higher levels of expenditure.

[5] In our calculations we set $\gamma = .98$, so utility is close to logarithmic.

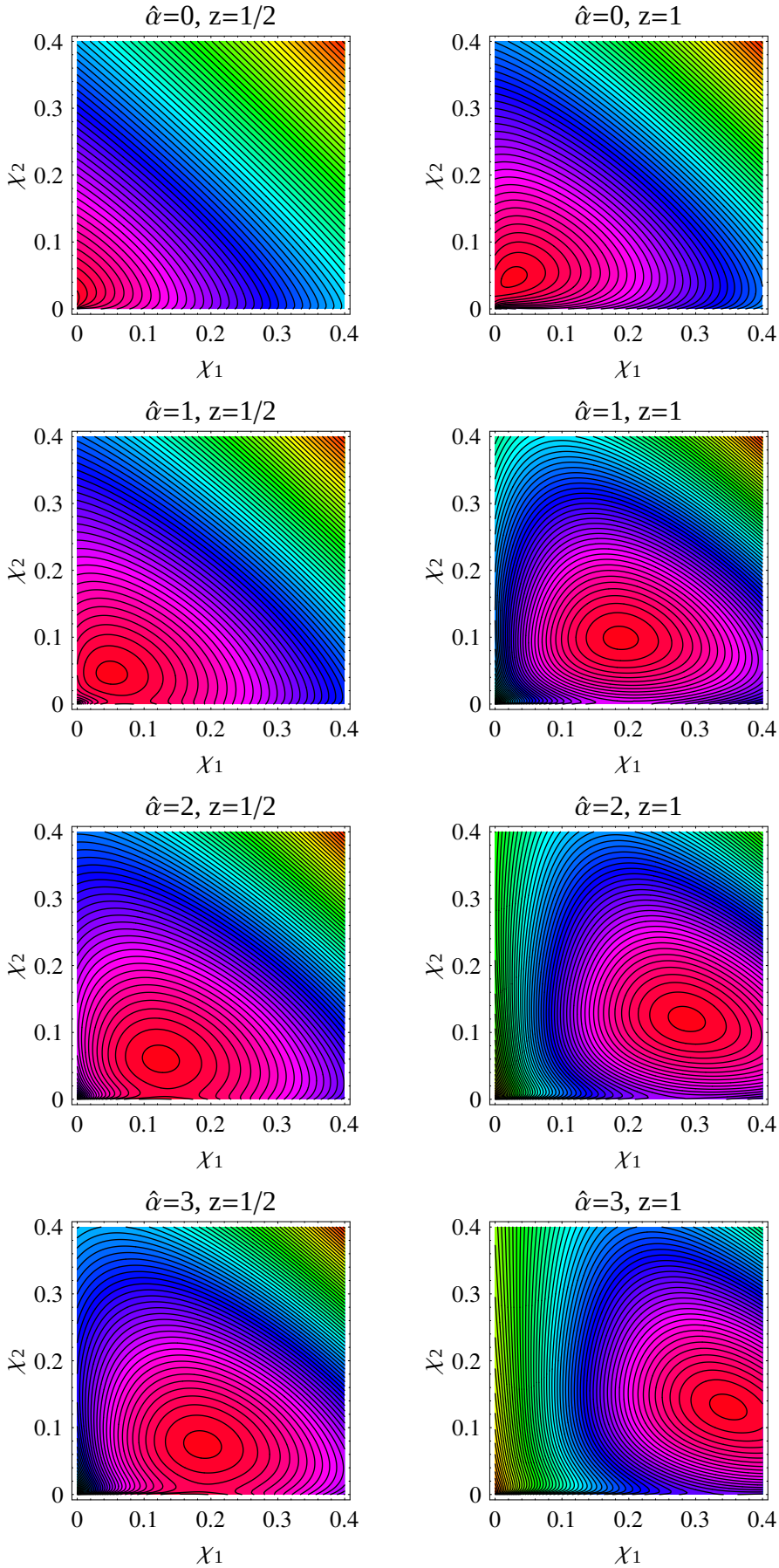

Figure 3: Contour plots for the value of $\widetilde{R}$ for different values of $\widehat{\alpha}$ and z.

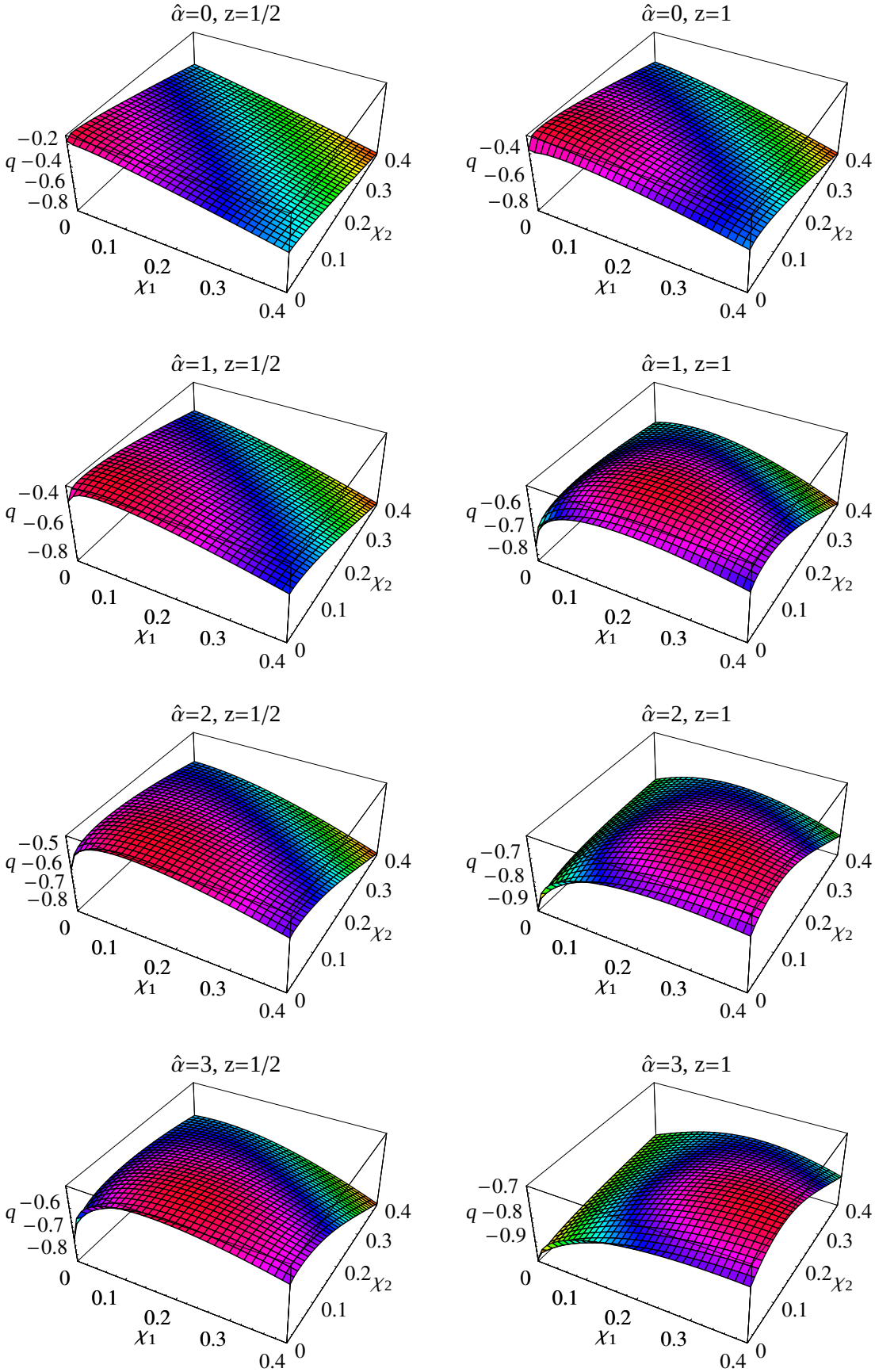

Figure 4: Three Dimensional plots for the value of $\widetilde{R}$ for different values of $\widehat{\alpha}$ and z. The value of $q = c_E - 1$.

	χ_1	χ_2	$\tilde{P}_W$	$\mathrm{E}(\psi \mid \chi)$	$\Pr\{\psi \geq .4z\}$	c_E
			$z = 1/2$			
$\widehat{\alpha} = 0$	0	0.015	0.671	0.172	0.378	0.861
$\widehat{\alpha} = 1$	0.047	0.051	0.480	0.180	0.397	0.670
$\widehat{\alpha} = 2$	0.125	0.061	0.382	0.176	0.373	0.558
$\widehat{\alpha} = 3$	0.188	0.075	0.318	0.166	0.329	0.482
			$z = 1$			
$\widehat{\alpha} = 0$	0.027	0.047	0.517	0.355	0.392	0.710
$\widehat{\alpha} = 1$	0.188	0.100	0.299	0.305	0.289	0.486
$\widehat{\alpha} = 2$	0.287	0.121	0.223	0.267	0.215	0.383
$\widehat{\alpha} = 3$	0.349	0.132	0.185	0.244	0.175	0.328

Table 2: Values of χ_1, χ_2, $\tilde{P}_W$, $\mathrm{E}(\psi \mid \chi)$, $\Pr\{\psi \geq .4z\}$ and c_E at the highest levels at which expected utility can be satisficed with robustness equal to 0, 1, 2 and 3, and z=1/2 and 1.

A comparison of the two different levels of defense expenditures and their allocations demonstrates that if a policy maker is interested in achieving some level of robustness against the unreliability of the probabilistic estimates for both damage and attack, he will opt to substantially raise the total amount of resources devoted to security. Furthermore, rather than increasing the allocations to each by either the same amount or the same proportions, he will choose an allocation skewed in favor of the more traditional type of expenditure, χ_1. The higher the required robustness, the more pronounced this property becomes. If required robustness increases to 200%, the total amount of defense expenditure is 40.8% of GDP, with 28.7% of GDP alloted to traditional military expenditures, χ_1, and 12.1% to RMA, χ_2. To achieve robustness at the 300% level, the policy maker must spend 48.1% on defense, devoting 34.9% to traditional armaments, and 13.2% to RMA-type expenditures.

One implication of the above example is that even achieving 100% robustness requires devoting a large fraction of the economy's resources to defense. However, this is the case when the parameter z, which determines the maximum amount of possible damage, is set to one, and there is some positive probability of complete annihilation.

Suppose instead we set $z = 1/2$, so that the greatest possible loss, in the event of war, will not exceed half of GDP; policy makers maximize expected utility by apportioning a meager 1.5% of GDP to defense, all of which is devoted to RMA-type expenditures. If policy makers choose to forgo robustness, they will completely abandon spending on the traditional military. If instead they demand robustness at a level of 100%, they will choose an allocation between traditional and RMA-type expenditures of 4.7% and 5.1% respectively. At a robustness level of 200%, the total expenditure rises to 18.6% and traditional expenditures, χ_1, more than double to 12.5%. At 300% robustness, total expenditure is 26.3%, more than two-thirds of which (18.8% of GDP) is spent on the traditional military.

In the last column of Table 2, we present the values of c_E, which represent the different levels of welfare, in terms of expected utility from consumption, associated with different combinations of robustness, ($\hat{\alpha}$) and defense expenditures, (χ_1 and χ_2). This measure is calculated as follows. First, we define $R_E(n)$ as the greatest value at which the expected utility can be satisficed, with robustness equal to n:

$$R_E(n) = \max_{\chi}\{R_c : \widehat{\alpha}(\chi, R_c) = n\} \tag{19}$$

The value of c_E is the consumption-equivalent to $R_E(n)$, defined through the utility function as: $R_E(n) = u(c_E)$. The contours in Figure 3, and the levels in its three dimensional counterpart Figure 4, represent the value of $q = c_E - 1$, the deviation in welfare (calculated in terms of equivalent consumption) from the utopian outcome associated with neither war nor any spending

on defense. The highest points in Figure 4 are attained by the values of χ_1 and χ_2 in Table 2.

The values of c_E in Table 2 demonstrate that achieving robustness entails some cost. For example, suppose the maximum level of damage equals 1/2, and the policy maker is willing to put his complete trust in the estimated damage probability distribution and the probability of war, $\alpha = 0$. Again, the policy maker devotes only 1.5% of GDP to defense, all alloted to RMA-type expenditures, and this generates a value of c_E=.861. Hence, the combined loss in welfare associated with the defense expenditure, and the random effects of these expenditures on the probability of war and its accompanying expected damage, are worth 14% of GDP. If on the other hand, the policy maker has less faith in the reliability of the information used to generate the probability distributions, insulating himself against deviations of up to 100% from both the estimated distributions necessitates raising overall defense spending to 9.8% of GDP, nearly equally divided between traditional and RMA-type expenditures. Achieving this level of robustness lowers the value of c_E to 0.67—pursuing a policy that is robust to model misspecification at the 100% level requires sacrificing an additional 19% of GDP, when compared to the defense allocation associated with zero robustness.

Achieving yet higher levels of robustness will entail even greater sacrifices however, the relationship is not linear but rather concave, so that raising robustness to 200% yields a c_E of 0.558—an incremental loss in welfare of 11.2% of GDP. Raising robustness still further, to 300%, lowers the value of c_E to 0.482, an incremental loss in welfare of only 7.6% of GDP.

4 Conclusion

Our application of the information gap approach to the problem of allocating resources for national security yields three broad conclusions, each of which is evaluated quantitatively. First, the higher the robustness to model misspecification policy makers demand, the higher the overall level of defense expenditures required. Second, the greater the demand for robustness, the greater should be the reliance on those defense measures better suited to prevent extremely high levels of damage. Similarly, the more robustness policy makers wish to achieve, the more they should eschew investment in systems that promise the best expected outcomes on the battlefield, but are also more vulnerable to catastrophical damage.

Beyond the normative recommendations for how policy makers can best allocate resources to protect their countries from aggression, in a world in which much of the relevant information is unreliable, the model also provides some positive insights into the reasons for the policies policy makers choose today or have chosen in the past. In a world with unreliable probabilistic information, we should not be surprised if policy makers lavish higher expenditure on defense than would be appropriate if the only goal were expected utility maximization. Furthermore, we

would expect policy makers to favor expenditures on weapons systems, and associated tactics and strategies, that are both most effective in preventing worst-case scenarios and are also better understood. These would suggest one possible *rationale* for military planners' reputation for conservatism, and indeed inertia, when confronted with new and untried technologies and doctrines.

5 Appendix

5.1 Specification of the Best Estimate of Damage, Conditional on Being Attacked

The best estimate of damage, conditional on being attacked is:

$$\widetilde{p}(\psi|\chi) = \frac{\psi^{a(\chi)-1}(z-\psi)^{b(\chi)-1}z^{1-a(\chi)-b(\chi)}\Gamma(a(\chi)+b(\chi))}{\Gamma(a(\chi))\Gamma(b(\chi))} I_{(z-\psi)} \tag{20}$$

where the Γ function is given by $\Gamma(x) = \int_0^\infty t^{(x-1)}e^{-t}dt$, $I_{(z-\psi)}$ is the indicator function and:

$$a(\chi) \quad = 1 + \tfrac{\chi_1}{\chi_2} + \theta e^{\theta\chi_1} \ln(\chi_1+\chi_2)\,\chi_1 \tag{21}$$

$$b(\chi) \quad = 2 + \tfrac{\chi_1}{\chi_2} + e^{\chi_2} \ln(\chi_1+\chi_2)\,\chi_2 \tag{22}$$

The mean of this pdf is:

$$\mathrm{E}(\psi|\chi) = z\frac{\Gamma(1+a(\chi))\Gamma(a(\chi)+b(\chi))}{\Gamma(a(\chi))\Gamma(1+a(\chi)+b(\chi))} \tag{23}$$

We set the value of $\theta = 3$ in our calculations.

5.2 Specification of the Best Estimate of the Probability of Being Attacked

The best estimate of the probability of being attacked is:

$$\widetilde{P}_{\mathrm{w}}(\chi) = 1 - \left(\frac{\beta_{\frac{1}{2}}(a(\chi),b(\chi))\Gamma(a(\chi)+b(\chi))(\chi_1+\chi_2)}{\Gamma(a(\chi))\Gamma(b(\chi))}\right)^{\frac{1}{4}} \tag{24}$$

where $\beta_x(a,b) \equiv \int_0^x t^{a-1}(1-t)^{b-1}dt$ and $a(\chi)$ and $b(\chi)$ are defined in (21) and (22).

The term $\chi_1+\chi_2$ is the total military expenditure, and reflects its deterrent value. The term $[\beta_{1/2}(a,b)\Gamma(a+b)]/[\Gamma(a)\Gamma(b)]$ is the probability of an enemy attack successfully inflicting at most half of the maximal potential damage. The higher this number is, the lower the likelihood that an adversary will be tempted to launch an attack.

5.3 Derivation of the Robustness Function

We derive the robustness function in (15) based on assumption (13) and for values of the robustness not in excess of unity: $\widehat{\alpha} \leq 1$. We make no assumptions about the utility function $u(c)$ other than that the marginal utility is positive: $\dot{u}(c) > 0$.

The robustness is defined in (5). The main task is to find the pdf of the damage, $p(\psi|\chi)$ which, at horizon of uncertainty α, minimizes the expected utility $R(\chi|p, P_{\mathrm{w}})$ defined in (1). Because the marginal utility is positive it is evident that $R(\chi|p, P_{\mathrm{w}})$ is minimized by that pdf in $\mathcal{U}(\alpha, \widetilde{p}, \widetilde{P}_{\mathrm{w}})$ which assigns as much weight as possible at large levels of damage and as little weight as possible at low levels of damage. For the fractional-error info-gap model in (11) one readily shows that $\min_{\alpha} R(\chi|p, P_{\mathrm{w}})$ occurs with the following pdf:

$$p(\psi|\chi) = \begin{cases} (1-\alpha)\widetilde{p}(\psi|\chi) & \text{if } \psi \leq \psi_{\mathrm{m}} \\ (1+\alpha)\widetilde{p}(\psi|\chi) & \text{else} \end{cases} \tag{25}$$

where ψ_{m} is the median of the estimated pdf $\widetilde{p}(\psi|\chi)$ and where $\alpha \leq 1$.

If $\alpha > 1$ then $\min_{\alpha} R(\chi|p, P_{\mathrm{w}})$ occurs with the following pdf:

$$p(\psi|\chi) = \begin{cases} 0 & \text{if } \psi \leq \psi_{\mathrm{s}} \\ (1+\alpha)\widetilde{p}(\psi|\chi) & \text{else} \end{cases} \tag{26}$$

where ψ_{s} satisfies:

$$(1+\alpha)\int_{\psi_{\mathrm{s}}}^{1} \widetilde{p}(\psi|\chi)\,\mathrm{d}\psi = 1 \tag{27}$$

In other words, ψ_{s} is the $1 - 1/(1+\alpha)$ quantile of $\widetilde{p}(\psi|\chi)$.

We will consider only the case $\alpha \leq 1$. The derivation of robustness in excess of unity is analogous.

The utility $R(\chi|pP_{\mathrm{w}})$ in (1), evaluated with the pdf in (25), is:

$$R(\chi|pP_{\mathrm{w}}) = [\widetilde{r} - \delta_r\alpha - u_{\mathrm{c}}]\,P_{\mathrm{w}} + u_{\mathrm{c}} \tag{28}$$

where $\widetilde{r}$ and δ_r are defined in (12) and (18) and u_{c} is the utility if an attack does not occur, $u(\chi_{\mathrm{c}})$. The term $\widetilde{r} - \delta_r\alpha - u_{\mathrm{c}}$ is negative so the minimizing value of P_{w} in $\mathcal{U}(\alpha, \widetilde{p}, \widetilde{P}_{\mathrm{w}})$ is $(1+\alpha)\widetilde{P}_{\mathrm{w}}$. Thus the minimum expected utility, up to horizon of uncertainty α, is:

$$\min_{p, P_{\mathrm{W}} \in \mathcal{U}(\alpha, \widetilde{p}, \widetilde{P}_{\mathrm{w}})} R(\chi|p, P_{\mathrm{w}}) = [\widetilde{r} - \delta_r\alpha - u_{\mathrm{c}}]\,(1+\alpha)\widetilde{P}_{\mathrm{w}} + u_{\mathrm{c}} \tag{29}$$

Denote this minimum $\mu(\alpha)$, which decreases monotonically as α increases because $\delta_r > 0$. The robustness is, according to the definition in (5), the greatest value of α up to which $\mu(\alpha)$ is no less than R_{c}. That is, the robustness is the lowest non-negative solution for $\widehat{\alpha}$ of:

$$\mu(\widehat{\alpha}) = R_{\mathrm{c}} \tag{30}$$

This is a quadratic equation in α whose least non-negative root is (15).

The derivation of the robustness function in (30) is obtained by equating (28) to R_{c} and solving for α, with $\widetilde{P}_{\mathrm{w}}(\chi)$ instead of P_{w}.

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