Games on Large Networks:

Approximate Equilibrium and Dynamic Rules

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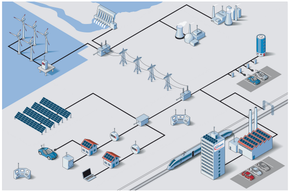
Introduction: Examples

- **Engineering Examples**: Distributed electricity production and consumption, Traffic Networks, etc ...



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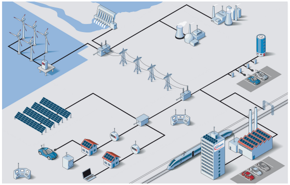
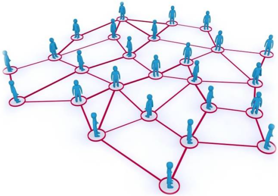


- **Social Networks**: Telecommunication company selection, Opinion about an idea or a product, Selection of fashion group, Engagement in criminal behavior, etc ...

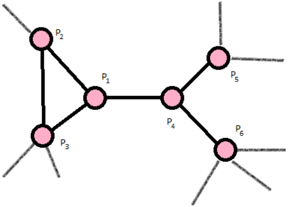


- **Non-Social Networks** Stores for renting, Gas Station Prices, ect...

Game Description: Dynamics and Costs

Interaction Structure:

- Set of players $(p_1, \ldots, p_N)$
- Each player p_i has a type $\theta_i \in \Theta$.
- Graph of interactions G.



Dynamics: $x^i_{k+1} = \sum_{j \in N^1_i(G)} f^{\theta_i \theta_j}(x^i_k, x^j_k, u^i_k, u^j_k, w^i_k)$

Cost Functions: $J_i = E\left\{\sum_{k=0}^{T} \rho^k \left[\sum_{j \in N^1_i(G)} g^{\theta_i \theta_j}(x^i_k, x^j_k, u^i_k, u^j_k)\right]\right\}$

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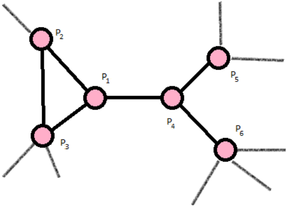

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Information:

- *Local*
- *Statistical*: The players consider a statistical ensemble of games

Approximate Equilibrium: Definition

Definition of an approximate equilibrium concept for ensembles of games.

Definition (ε - Probabilistic Approximate Nash Equilibrium)

Consider an ensemble of Interaction Structures $\mathcal{E}$. A set of strategies $(\gamma_i)_{i=1}^N$ is $\boldsymbol{\varepsilon}$ - **Probabilistic Approximate Nash Equilibrium** for the ensemble $\mathcal{E}$ if it holds:

$$P(\{S \in \mathcal{E} : (\gamma_i)_{i=1}^N \text{ is an } \varepsilon - \text{Nash equilibrium}\}) > 1 - \varepsilon$$

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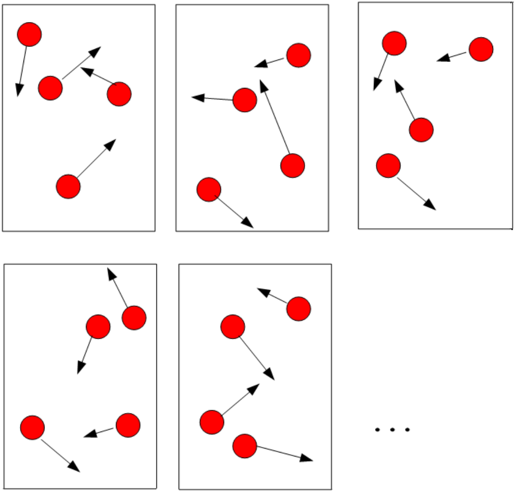
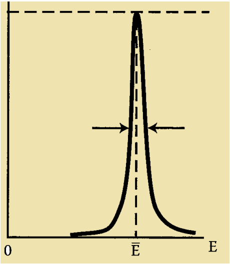
- How much information is needed to have a PAN equilibrium.
  Complexity Functions

Approximate Equilibrium: Statistical Physics Analog



The total energy in the canonical ensemble



Special Cases

Examples of games with low complexity:

- Static or LQ games on *Erdos-Reyni* random graphs or *Small World* nets: **Law of large numbers**
- Static game on *Lattices* : **Contraction mapping ideas**
- LQ game on a *Ring*; **Low gain to distant players**
- A non-quadratic static game on a *ring*: **Cooperation among the players**

Dynamic Rules

What if there is not enough information?

- Nash Equilibrium
  - Stochastic Adaptive Control problem (Dual Control Problem): The actions of each player affect her own future estimation
  - The actions of each player affect the future estimation of the other players (like the Witsenhausen's counterexample)

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Examples:

- Adaptive Control Laws
- Learning
- Best Response
- Evolutionary Dynamics

The Cheating Problem

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1. When can a Dynamic/Adaptation rule serve as a prediction of the behavior of the players? **Assessment**

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1. When can a Dynamic/Adaptation rule serve as a prediction of the behavior of the players? **Assessment**
2. Are there any simple cheating strategies?
3. What are the outcomes of the game when (all or some of) the participants are cheating?

Pretending

Simple cheating strategy: **Pretending**

1. Game outcomes **alternative** to the Nash equilibrium.
2. Interesting relationships between **pretending** and **leadership**
3. Pretending may enhance **cooperation**, **competition** or even make a system designed to work well on the Nash equilibrium **not working at all**

Future Directions

- Time varying network topologies
- Develop testable conditions to assess the adaptive laws and develop laws less sensitive to cheating
- Network design to reduce the ability to cheat