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ABSTRACT

This article studies the cost function for the natural gas transmission industry. In addition to a tribute to H.B. Chenery, it firstly offers some further comments on a recent contribution (Yépez, 2008): a statistical characterization of long-run scale economies, and a simple reformulation of the long-run problem. An extension is then proposed to analyze how the presence of seasonally-varying flows modifies the optimal design of a transmission infrastructure. Lastly, the case of a firm that anticipates a possible random rise in its future output is also studied to discuss the optimal degree of excess capacity to be built into a new transmission infrastructure.

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Introduction

As far as engineering economics is concerned, the year 2009 corresponded to a special anniversary: 60 years ago, Hollis B. Chenery, a then promising PhD student – he later became the World Bank's Vice President for Development Policy – published a seminal article that illustrated how the production function of microeconomic theory can be rewritten with engineering variables (Chenery, 1949). His goal was to present a rigorous analysis of the cost function of an industry whose total production function consists of several processes which can be combined in varying proportions to produce a given output. As an illustration, he provided an illuminating case study based on the natural gas transmission industry. Analyzing the design of a simple infrastructure – a compressor station and a pipeline – he remarked that the combination of processes is such that it is possible to increase output by adding more compressors while keeping the same pipeline or, in the long run, simultaneously varying both parameters. As he has shown, this flexibility is the origin of the massive scale economies observed in that industry. Despite a considerable early influence in the academic community (e.g.: Smith, 1957, 1959; Thomson et al., 1972), the engineering economist approach pioneered by Chenery has gradually disappeared from the applied economic literature. Apart from rare exceptions like Callen (1978), most of the economic and/or policy articles dedicated to the natural gas industry did not consider this engineering approach. Given the impressive regulatory reforms implemented in that industry during the last decades, this fall into oblivion is somehow astonishing.

On the engineering side, Chenery's ideas have also infused the operations research literature. During the last decades, the problems associated with the construction and subsequent management of large transmission networks have offered a stimulating field of research. Many models have thus been developed: to define the optimal operation of an already developed infrastructure (De Wolf and Smeers, 2000), or to identify the optimal expansion plans of that infrastructure (Kabirian and Hemmati, 2007; André et al., 2009), or to choose the optimal long-run design for a network whose topology is given (Hansen et al., 1991; De Wolf and Smeers, 1996; Ruan et al., 2009). Besides these numerical models, strong analytical results have also been obtained for special cases. For example, André and Bonnans (2010) studied the optimal design of a "trunkline system" (i.e. a long-distance, wide-diameter pipeline that has several compression stations installed along the pipe) and proved that the optimal design of these infrastructures involves regularly spaced compression stations (except perhaps for the last one), a unique pipe-diameter for the whole system (except perhaps for the last pipe element) and equal compression ratios for the compressors (except perhaps for the last one). Clearly, the technical sophistication of these contributions cannot be compared with Chenery's simple setting. Nevertheless, these articles share an obvious intellectual relation with Chenery (1949) and it is somehow unfortunate that this proximity has unintentionally been forgotten.

Hopefully, a recent article published in this Journal (Yépez, 2008) has provided a refreshing revival of Chenery's ideas. In addition to the perpetuation of this method, at least three reasons can be advanced to illustrate the value added by this contribution. Firstly, the paper presents a clear and modernized version of Chenery’s methodology which includes some interesting engineering refinements, such as a more recent version of the gas flow equation, and the possibility to deal with a non-steady elevation of the pipeline. Secondly, Yépez derived a set of analytical equations that describe how total, average and marginal costs vary with output in the short run. And last but not least, Yépez provided a detailed case study based on a Mexican project. As this presentation of Chenery (1949) only represents the tip of an iceberg, some complements may be needed to shed some light on Chenery's shrewd thoughts on the gas transmission industry.

This article is organized as follows. In the first section, a complete review of the Chenery-Yépez model aims at offering some eclectic complements. In the second section, an extension of that model is provided to deal with the case of seasonal variations in the volumes of gas to be transported. The influence of these variations on the optimal design of the transmission equipment is discussed. Investment recommendations are then addressed in the last section to analyze the rationale of a “building ahead of demand” behavior.

1 – A commented review of the Chenery-Yépez approach

In this first section, three points are successively discussed: the long-run optimal decision, the short-run one and a complementary remark on an alternative formulation of the long-run problem.

1.1 Long-run economics

Both authors developed a rigorous model-based cost function for a simple gas transmission infrastructure: a compressor pumps a daily flow of natural gas Q into a pipeline that runs a given distance l.

In Chenery (1949) and Yépez (2008), output and investment decisions are assumed to be taken separately and the estimate of output is supposed to be made prior to the investment decision. This assumption is consistent with industrial practice because, in many cases, this flow is an outcome of exogenous negotiations between a natural gas producer and a group of buyers. As the firm is assumed to have perfect information on the volume of gas to be transported and on the associated revenues, the project's value is maximized by an investment program which minimizes the total cost of production over the period. Since output is assumed to remain steady all through the infrastructure lifetime, the optimum plan also minimizes the annual total costs. In this long-run decision, two engineering variables are considered: D the inside diameter of the pipe, and H the compressor horsepower.

Production function

Any combination of inputs (D, H) must be compatible with the technological constraints faced by the gas transmission firm. In that industry, two engineering relations must simultaneously hold.

The first relation is a flow equation that gives the frictional loss of energy through the pipe – measured as pressure drop between the inlet pressure P_1, and the outlet one P_2 both in psia (pounds per square inch absolute) – obtained while piping a given steady flow rate Q in MMcfd (million cubic feet per day) across a distance l (in miles) on a pipe whose inside diameter is D (in inches). A flow equation typically requires two things: the general flow equation derived from thermodynamic reasoning, and an empirically-determined friction coefficient (Mohitpour et al., 2003; Shashi Menon, 2005). Several empirical models (Weymouth, Panhandle, AGA...) have been proposed to determine the value of that friction parameter as a function of engineering variables (e.g.: pipe diameter, Reynolds number...) and gas flow-regimes (laminar, partially turbulent, turbulent). Hence, gas engineers have to arbitrarily choose a model. For a high-pressure infrastructure that must transport a given high flow rate gas with a given pressure drop, the Weymouth model is known to provide a reasonably conservative value for the pipe-diameter (Mohitpour et al., 2003; Shashi Menon, 2005). Because of safety concerns, this model is usually recommended for pipe design decisions (Mohitpour et al., 2003 p. 79). That's why, the Weymouth equation has been assumed in this study. This choice is consistent with Chenery (1949) and Yépez (2008). With the assumption that there is no elevation change in the pipeline and that the terminal pressure P_2 is an exogenously determined parameter, the Weymouth flow equation is (Mohitpour et al., 2003; Shashi Menon, 2005):

$$Q = \frac{c_0}{\sqrt{l}} D^{8/3} \sqrt{\left(\frac{P_1}{P_2}\right)^2 - 1}, \tag{1}$$

where c_0 is an exogenous constant parameter.

The second relation gives the power required to compress natural gas from a given inlet pressure P_0 to a predefined outlet pressure P_1 (Yépez, 2008):

$$H = c_1.\left(R^{\beta} - 1\right)Q, \tag{2}$$

where H is the horsepower per million cubic feet of gas, R is the pressure ratio $P_1/P_0 \geq 1$ and both c_1 and β are positive dimensionless constant parameters with $\beta < 1$.

For the sake of simplicity, the terminal pressure P_2 at the delivery point is assumed to be equal to the inlet one P_0 as in Chenery (1949). These two technological equations can then be simplified (by eliminating R) into a single engineering production function that has to be equal to zero:

$$F(D,H,Q) = \frac{lQ^2}{{c_0}^2 D^{16/3}} + 1 - \left(\frac{H}{c_1 Q} + 1\right)^{\frac{2}{\beta}} = 0\,. \tag{3}$$

This production function embodies the key features of the gas transmission industry and suggests the possibility of a smooth continuous substitution between pipe-diameter and compression horsepower. From a strict technical perspective, the combination of these two capital factors (diameter and compressor horsepower) is such that it is possible to increase output by adding more compressors while keeping the same pipeline or increasing the pipeline diameter while keeping the same design for the compressors.

Annual costs

As far as costs are concerned, two different elements have to be considered. Firstly, the total yearly capital and the operating cost per mile is given by a smooth function of the inside diameter D and the pipe thickness τ. Following Chenery (1949) and Yépez (2008), τ is assumed to be a linear function of the pipe diameter which allows a univariate expression of the pipeline cost. With Yépez's terminology, this cost function is named $C_D(D)$ and is equal to $\alpha_L.C_1(D)+C_2(D)$, where $C_1(D)$ is the replacement value of line per mile dependent on the diameter; α_L is the fixed-cost annual percentage charge dependent on the depreciation and real interest rates so that $\alpha_L.C_1(D)$ gives the annual cost of the line per mile, and $C_2(D)$ the annual operation and maintenance cost per mile.

The annual cost of the compressor station $C_H(H)$ constitutes the second type of cost. Again: $C_H(H)$ is the sum of two smooth functions of H the horsepower: $\alpha_C.C_3(H)$ and $C_4(H)$, where α_C is the fixed-cost annual percentage charge dependent on the depreciation and real interest rates, $C_3(H)$ is the replacement value of the compressor station dependent on the installed horsepower capacity and $C_4(H)$ is the operation and maintenance cost.

Obviously, the choice of functional specifications can play a non-negligible role on gas transmission economics. Hence, they deserve a short discussion as we notice some differences in the literature. In Chenery (1949) and in most operation research studies, a linear specification has been assumed for these annual cost functions. Yépez (2008) makes a different choice as these annual costs $C_D(D)$ and $C_H(H)$ are assumed to be strictly increasing, smooth and concave functions of the indicated engineering variable (the diameter or the compression horsepower). The resulting unit costs of the pipeline and compressor inputs are thus assumed to decrease with the indicated engineering variable (see Appendix 1 for a summary and a discussion on these cost assumptions).

Problem formulation

For a rational firm that must transport a given flow Q, the objective is to find a technologically-compatible combination of inputs that minimizes its annual total costs. Hence, the firm's long-run total cost function $LRTC(Q)$ can be defined as follows:

$$LRTC(Q) = \begin{cases} \underset{D,H}{\text{Min}} & l.C_D(D) + C_H(H) \\ \text{s.t.} & F(D,H,Q) = 0. \end{cases} \tag{4}$$

With the functional specifications and the numerical parameters presented in Yépez (2008), this constrained minimization problem has a unique solution. This optimal mix of inputs (D^*, H^*) can be obtained thanks to the Lagrangian method. The Lagrangian function $\pounds$ for this constrained minimization problem is:

$$\pounds(D,H,\lambda) = l.C_D(D) + C_H(H) + \lambda\, F(D,H,Q). \tag{5}$$

And the optimal solution (D^*, H^*) satisfies the first-order necessary conditions:

$$\frac{\partial \pounds}{\partial D}(D,H,\lambda) = l.C_D^{'}(D) + \lambda\, \frac{\partial F}{\partial D}(D,H,Q) = 0 \tag{6}$$

$$\frac{\partial \pounds}{\partial H}(D,H,\lambda) = C_H^{'}(H) + \lambda\, \frac{\partial F}{\partial H}(D,H,Q) = 0. \tag{7}$$

Where, $C_D^{'}(D)$ (respectively $C_H^{'}(H)$) is the marginal annual cost of the pipeline (respectively of the compressor station). Note that a straightforward property of that optimal combination of pipeline diameter and the horsepower of the compressor station (D_{LR}^*, H_{LR}^*) can be exhibited. These two first-order conditions imply that the optimum combination (D_{LR}^*, H_{LR}^*) satisfies:

$$\frac{l.C_D^{'}\left(D_{LR}^{*}\right)}{C_H^{'}\left(H_{LR}^{*}\right)} = \frac{\frac{\partial F}{\partial D}\left(D_{LR}^{*}, H_{LR}^{*}, Q\right)}{\frac{\partial F}{\partial H}\left(D_{LR}^{*}, H_{LR}^{*}, Q\right)}. \tag{8}$$

Scale economies, some empirical evidences

In his discussion on the long-run economics of the natural gas transportation industry, Yépez noticed the presence of significant economies of scale. His affirmation was inspired by the shape of the long-run cost curves as both long-run average cost (LRAC) and long-run marginal cost (LRMC) are decreasing and the former exceeds the later. To complete this plot-inspired remark, some additional results may be useful to quantify those scale economies. In another paper, Chenery suggested that the long-run cost function of the gas pipeline has an almost constant elasticity of output with respect to cost over most of its range (Chenery, 1952). Thus, an acceptable approximation of the long-run total annual cost function $LRTC(Q)$ may be provided by the function $\gamma Q^{1/\psi}$, where γ is a constant and ψ represents a scale coefficient (with that specification, the ratio of average cost to marginal cost is constant and equal to ψ). Given that Yépez's analysis differs from Chenery (1949) (as it includes a modernized version of the technological relations and a non-linear specification for the input cost functions $C_D(D)$ and $C_H(H)$), the validity of Chenery's approximation needs to be validated with a statistical approach.

Hence, an investigation based on Griffin's "pseudo data" method seems needed (Griffin, 1977, 1978, 1979). This method was developed in the 1970s. At that time, available computational technologies prohibited the direct inclusion of cumbersome detailed engineering models in large inter-industry simulation models. The "pseudo data" approach was seen as a computational-friendly tool able to simplify complex process models into single-equation cost functions. Each of these cost functions depicted the long-run total cost for various quantities and was estimated from a data set generated from the associated industry process model. In the present study, a data set of 84 observations was generated by running several instances of Yépez's model with the list of parameters presented in Appendix 1. In these numerical simulations, the output varied from 0.25 to 21 Bcm/year (billion cubic meters of gas per year) – i.e. 25.6 to 2147.5 MMcfd – by regular steps of 0.25 Bcm/year. Here, cost data were drawn over a sufficiently wide range that represents usual operation conditions in the natural gas transmission industry. Based on these "pseudo data", simple relationships can be statistically estimated. Thanks to the usual log transformation, an Ordinary Least Square (OLS) regression is sufficient to estimate the specification suggested in Chenery (1952). Table 1 presents the empirical results for this statistical cost function.

Table 1: Empirical results obtained from an OLS regression on Chenery's specification. The numbers in parentheses are standard errors of coefficients.

$\log(LRTC)$	=	12.34312	+	0.613467	$\log(Q)$
		(0.000984)		(0.000145)	
R^2 = 0.999995 ; S.E. of regression = 0.001218					

The goodness of fit measure R^2 indicates an excellent explanatory power which is quite unusual for such a simple specification. The estimated values are highly significant as the t-statistics suggest that the probabilities attached to the real values being 0 are insignificant. Hence, Chenery's specification provides an acceptable approximation of the long-run total cost function.

As far as the long-run scale coefficient ψ is concerned, these findings confirm the presence of significant scale economies: for any given output, the Long-Run Average Cost is always 63% greater than the Long-Run Marginal Cost. This result undoubtedly reinforces the pertinence of Yépez's conclusion on the design of appropriate pricing policies: a long-run marginal-cost pricing would not allow the firm to break even (Yépez, 2008).

1.2 Short-run cost economics

In the short-run, there are some technical and economical restrictions on the substitutability between the two factors D and H. As suggested by Chenery (1952), it seems reasonable to consider any already installed pipeline as an indivisible factor. Following Yépez (2008), we thus assume that the short-run adjustments needed to serve an additional demand are obtained by increasing the compression horsepower while keeping the pipeline unchanged.

For a given pipeline whose diameter is D, the engineering production function (3) can be reorganized to define a short-run factor demand function $H = g_D(Q)$ that gives the amount of compression horsepower H required to propel the daily flow rate of natural gas Q. Here, g_D is a smooth, strictly increasing and strictly convex function:

$$g_D(Q) = c_1 Q.\left(\left(\frac{lQ^2}{c_0^{\ 2} D^{16/3}} + 1\right)^{\frac{\beta}{2}} - 1\right). \tag{9}$$

With this technology, it is clear that a 100% increase in the quantity to be transported through a given pipeline requires a more than 100% increase in the compression horsepower.

Using (9), Yépez (2008) defined a single variable expression for the Short-Run Total Cost function $SRTC(Q)$ that includes a significant fixed pipeline cost and an output-variable compressor cost:

$$SRTC(Q) = l.C_D(D) + C_H(g_D(Q)). \tag{10}$$

From (10), single variable expressions can easily be obtained for the Short-Run Average Cost function $SRAC(Q)$ and the Short-Run Marginal Cost function $SRMC(Q)$. As a brief comment on the short-run economics of that industry, it may be interesting to underline that the Short-Run Average Cost function $SRAC(Q)$ has the usual "U-shaped" curve as illustrated in Figure 1. A formal proof of this affirmation is straightforward in the case of Chenery's linear compressor cost function. With the functional specification and numerical parameters used in Yépez (2008), my investigations also confirmed the presence of a "U-shaped" curve.

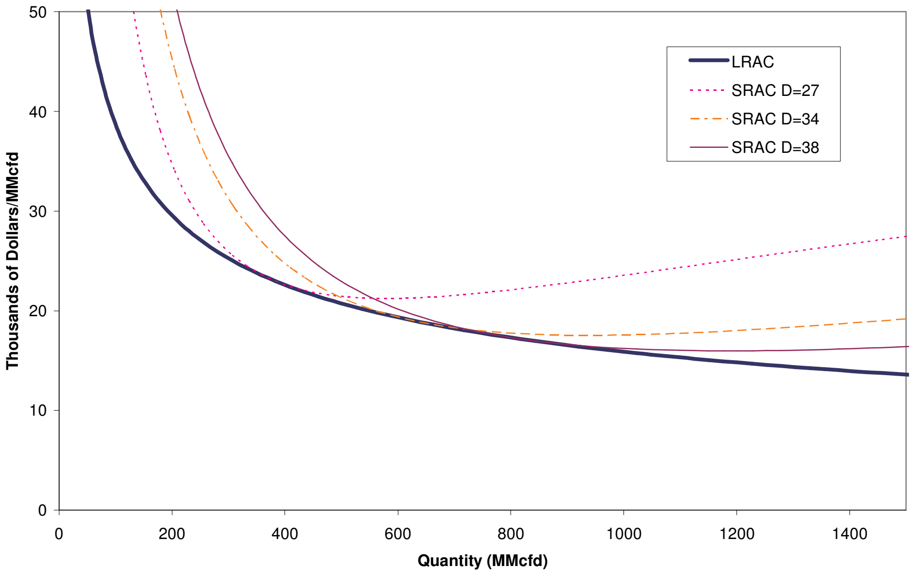

Figure 1: U-shaped Short-Run Average Cost curves obtained with various pipe-diameters.

1.3 Long-run vs. short-run, an alternative and simplified view

This subsection aims at showing that the long-run problem studied by both Chenery and Yépez can be reformulated in a simplified manner.

First, attention is focused on the role played by the pipe-diameter in the short run. Any change in that diameter has a major impact on the short-run economics of the transmission equipment since it changes the repartition of the total cost between fixed costs (pipeline related costs) and output-dependant ones (the compression costs). In some sense, the pipe-diameter can be viewed as an index of scale that characterizes the size of an installed transmission equipment (Chenery, 1952).

For a given flow rate of gas to be transported Q, the engineering production function (3) can also be reorganized to get the compression horsepower H required in the short run as a function of the pipeline diameter D (provided that D remains strictly positive):

$$f_Q(D) = c_1 Q.\left(\left(\frac{lQ^2}{c_0{}^2 D^{16/3}} + 1\right)^{\frac{\beta}{2}} - 1\right). \tag{11}$$

Because of the technical substitutions mentioned earlier, f_Q is a differentiable strictly decreasing function of the pipe-diameter.

The influence of the pipe-diameter D on the Short-Run Total Cost to transport a given flow rate of gas Q is described by the cost function parameterized by Q:

$$SRTC_Q(D) = l.C_D(D) + C_H\left(f_Q(D)\right). \tag{12}$$

Obviously, the pipeline element $l.C_D(D)$ is usually a strictly increasing function of the pipe-diameter, whereas the annual compression cost $C_H\left(f_Q(D)\right)$ is strictly decreasing with respect to D.

A simple reformulation of the long-run cost minimization problem can now be proposed. Each strictly positive value of D corresponds to a unique Short-Run Total Cost function that is parameterized by a continuous parameter: Q. Hence, a family of Short-Run Total Cost functions indexed by a continuous variable D can be defined. For a given output Q, each of these functions provides a short-run total cost and those values vary with the pipeline diameter D. In this subsection, it will be proven that the long-run cost minimization problem can be viewed equivalently as selecting an appropriate (and unique) element in that family of short-run cost curves.

Restated with simple algebra, the goal is to find the particular Short-Run Total Cost function that minimizes the annual cost of transporting a given flow Q of gas. As D is assumed to be a continuous parameter, a necessary condition for minimum annual cost is that the derivative of Short-Run Total Cost with respect to D be zero:

$$\frac{dSRTC_Q}{dD}(D) = 0. \tag{13}$$

Equivalently,

$$l.C_D^{'}(D) = -f_Q^{'}(D) \times C_H^{'}\left(f_Q(D)\right). \tag{14}$$

As usual, an economic interpretation can be given to that expression: at the optimum, the marginal cost of the pipeline is exactly equal to the marginal compression horsepower economy.

From a technical perspective, some straightforward arguments can be given to illustrate the existence of a unique solution. As $SRTC_Q(D)$ is differentiable with $\lim_{D\to 0^+} SRTC_Q(D) = +\infty$ and $\lim_{D\to +\infty} SRTC_Q(D) = +\infty$, it is clear (Rolle's theorem) that there is at least one solution to equation (13). Of course, the uniqueness of that solution depends on the functional specifications chosen for both C_D and C_H. With Chenery's linear specifications, the function $SRTC_Q(D)$ is strictly convex and thus has a unique global minimum. With the concave specifications defined in Yépez (2008), convexity is no longer verified but the numerical values used in Yépez (2008) insure a U-shaped curve with the succession of two strictly monotonic patterns: a decreasing one and an increasing one. As a result, there is also a unique minimum cost with Yépez's specification. Hence, the problem at hand is a well-behaved one. Let's denote D_{SR}^* that unique solution to (13).

It is now time to compare D_{SR}^* with those denoted $\left(D_{LR}^*, H_{LR}^*\right)$ obtained when solving the long-run cost minimization programme proposed by Chenery and Yépez. Here comes an interesting lemma:

> ***Lemma 1:*** *Assume a gas transmission firm with the costs and engineering production function as described above. For that firm, the long-run optimal combination of inputs* $\left(D_{LR}^*, H_{LR}^*\right)$ *that solves Yépez's cost-minimizing program is exactly equal to* $\left(D_{SR}^*, f_Q\left(D_{SR}^*\right)\right)$*, those obtained with the single variable problem above.*

For the interested reader, a straightforward proof is provided in Appendix 2.

This approach simplifies the long-run problem as it only requires solving a single variable equation. In the following sections, that simplification will be helpful to illustrate some new results. Besides, this method provides a clear argument for presenting the Long-Run Total Cost curve as the lower envelope of the Short-Run Total Cost curves (see Figure 2 for an illustration).

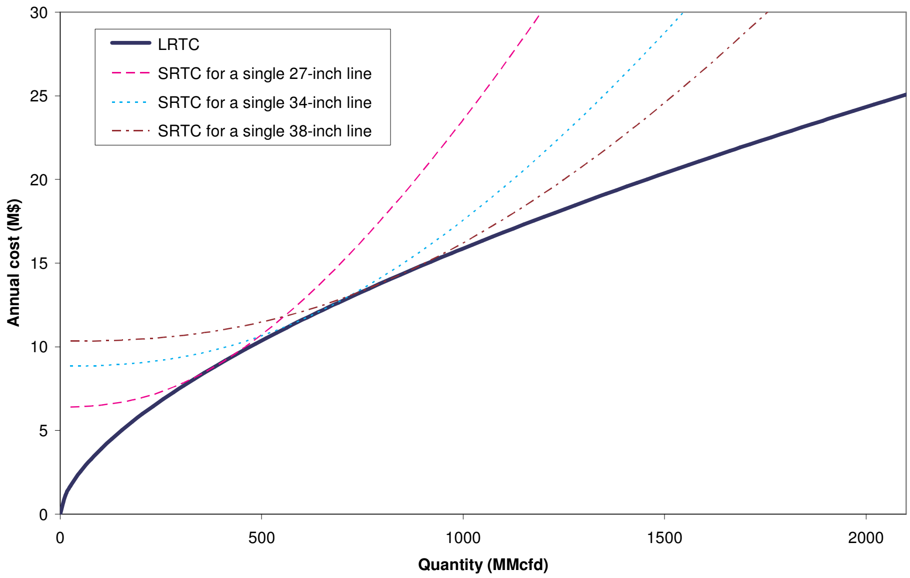

Figure 2: The Long-Run Total Cost curve as the lower envelope of the Short-Run Total Cost ones.

2 – Dealing with seasonal variations

Chenery (1949) and Yépez (2008) studied the case of transmission equipment that is planned to transport a steady flow of natural gas Q all through the year. Obviously, gas consumption varies over time – across seasons, weeks and days. Moreover, the amplitude of these variations can be large. A typical illustration is provided by the seasonal swing observed in countries where natural gas is largely used for heating. For example, in north-western Europe approximately two-thirds of the gas is consumed during the winter period (October–March). Furthermore, *"residential users consume about 90 percent of their overall gas during the winter period. For local gas providers, it is therefore not uncommon to have daily peaks in gas delivery in the winter amounting to more than ten times the delivery on a summer day."* (Höffler and Kübler, 2007, pp. 5206-5207). In many cases, gas cannot be stored near end-users. Significant seasonal fluctuations can thus be observed in the daily flows of gas to be transported.

How is the short-run total annual cost impacted by these seasonal fluctuations? How does this seasonal pattern influence the optimal design of an infrastructure? These are precisely some of the questions addressed in this second section.

2.1 A further distinction: expansion vs. contraction costs

This preliminary subsection provides a useful piece of methodological background. Following the usual convention, Yépez (2008) defines the short-run as a period of time in which the quantity of at least one input is fixed (here, pipe-diameter D) and the quantities of the other input H can vary. As in classic microeconomic theory, the long-run corresponds to a hypothetical situation in which the quantities of all inputs can vary. Aside from that usual distinction, a further distinction could be valuable for analyzing the short-run economics of that industry: that of expansion and contraction costs. That distinction, pioneered by the "French marginalist school", relies on the asymmetry between plant expansion and plant contraction since some fixed costs incurred in case of expansion cannot be recouped in case of contraction (Cf. Dreze, 1964, for a comprehensive survey). In fact, Chenery also perceived the pertinence of that further distinction: "*The functions resulting from fixing the pipe size (scale of plant) and varying other factors (amount of pumping equipment, etc.) will be called "intermediate" cost functions. It is possible to move along the intermediate cost curves only as long as demand is expanding. A contraction of demand will involve a movement along a "plant" curve (...) where the only important variable is fuel consumption.*" (Chenery, 1952, p. 4).

The framework presented in Yépez (2008) provides an interesting starting point to implement that distinction in the natural gas industry. On the assumption that the pipe-diameter remains unchanged in the short run, the output-variable element corresponds to the total annual cost of the compressor stations that includes the capital costs and the operation and maintenance costs. Given that any expansion requires the installation of an additional compression capacity whose annual cost is fully captured in this short-run cost function, it is clear that Yépez's short-run total cost function describes the variation of annual cost if output always increases. But in case of a sudden and temporary drop in output, rigidity would probably be observed in the downward adjustment of the compression capacity. Stated differently, it means that there are few chances for such a restriction in output to be accompanied with an instantaneous premature scrapping of the excess in compression capacity.

With those remarks in mind, and using the previous notations, a simple reformulation can now be proposed to distinguish between expansion and contraction costs. For a gas transmission infrastructure that transports a given flow Q_0 with a pipeline whose diameter is D and an adapted compression capacity $H_0 = g_D\left(Q_0\right)$, the total expansion cost incurred to serve a larger steady flow-rate of natural gas $Q \geq Q_0$ is given by Yépez's short-run total cost function $SRTC\left(Q\right)$ that includes the extra capital expenses required for the installation of an additional pumping capacity. In case of a contraction in the output to $Q < Q_0$, the annual compression station cost $\alpha_C.C_3\left(H_0\right)$ remains unchanged whereas the operation and maintenance costs are reduced to reflect a lower annual rate of operation of the compressors.

To summarize, the short-run total expansion cost function $SRTEC$ of that infrastructure is given by:

$$SRTEC(Q) = l.C_D(D) + \alpha_C.C_3(g_D(Q)) + C_4(g_D(Q)), \qquad \forall Q \geq Q_0 \tag{15}$$

Whereas the short-run total contraction cost function $SRTCC$ is:

$$SRTCC(Q) = l.C_D(D) + \alpha_C.C_3(H_0) + C_4(g_D(Q)), \qquad \forall Q < Q_0 \tag{16}$$

Hence, the short-run total cost function takes either one or the other expression depending on whether Q the output to be served, is larger or not than the initial level Q_0. As far as short-run marginal cost is concerned, we notice a discontinuity for the particular output $Q = Q_0$. For this output, the right derivative, i.e. $\frac{d}{dQ}\left[\alpha_C.C_3(g_D(Q)) + C_4(g_D(Q))\right]$, gives the marginal expansion cost whereas the left derivative, i.e. $\frac{d}{dQ}\left[C_4(g_D(Q))\right]$, corresponds to the marginal contraction cost. Explicit reference must thus be made to one of these concepts when considering practical applications of marginal costs. More generally, the possible non-equality of left-hand and right-hand short-run marginal costs at adapted capacity has already been emphasized in the literature (see for example Pierru, 2007, for an economic interpretation in the case of linear-programming models).

2.2 The case of seasonal variations

As I have pointed out, gas consumption varies with the seasons, weeks and days. To analyze the influence of these seasonal variations, we study the case of a rational firm that plans to build an equipment to transport time-varying gas flows. Here, a daily time scale is assumed and the analysis concentrates on the between-day variability in the gas flow rate. A steady flow regime is thus assumed within each day. This analysis could arguably be adapted to a different time unit provided that the duration of this time unit remains large enough compared to those of the transient periods during which an unsteady-state gas flow is observed in the pipe (because the modelled flow equation is only granted for steady-state gas flows). So, the firm is supposed to know q_t the future daily flow of gas demanded on day t, and Q_p the peak flow to be transported on that infrastructure (i.e. : $Q_p = \underset{t}{Max}\ q_t$). A one-year periodicity is assumed for the gas flows (i.e.: $q_t = q_{t+365}$). Given that a 100% load factor corresponds to the steady flow case studied above, a strictly less than one load factor is assumed for that infrastructure, meaning that there is at least a day t with $q_t < Q_p$. In this case, the compressor horsepower varies from one day to another and the horsepower required on day t is named H_t. During that day, the associated operation cost of the compressor is assumed to be equal to

1/365th of $C_4\left(H_t\right)$ the operation cost of the compressor that would be observed in case of a steady operation at H_t all along the year.

Here again the firm's decision can be analyzed as an annual cost-minimization problem. As in the Chenery-Yépez approach, the problem faced by the firm can be viewed as a cost-minimization program:

$$\begin{aligned}
&\underset{D,H_p,\{H_t\}_t}{\text{Min}} && l.C_D\left(D\right)+\alpha_C.C_3\left(H_p\right)+\frac{1}{365}\sum_{t=1}^{365}C_4\left(H_t\right) && \\
&\text{s.t.} && F\left(D,H_t,q_t\right)=0, && \forall t\in\{1,...,365\}, \\
& && H_t\leq H_p, && \forall t\in\{1,...,365\}.
\end{aligned}\tag{17}$$

The objective is to minimize the annual total cost incurred to transport the daily flows $\{q_t\}_{t\in\{1,...,365\}}$ with an equipment whose compressor station has a capacity H_p and whose pipeline has an internal diameter D. Of course, H_p must imperatively be large enough to provide any of the compression horsepower $\left\{H_t=f_{q_t}\left(D\right)\right\}_{t\in\{1,...,365\}}$. Given that the model at hand is fully deterministic, there is no incentive to build any extra capacity and the compression capacity to be installed is thus supposed to be equal to the minimum required to serve those peak flow Q_p. Hence, the peak compression horsepower H_p is given by $f_{Q_p}\left(D\right)$.

Once again, this problem can be reformulated as a single variable optimization programme: i.e. finding the unique optimal diameter $D^*_{\{q_t\}}$ that minimizes the short-run total contraction cost function $SRTCC$ to serve the flows $\{q_t\}_{t\in\{1,...,365\}}$ knowing that this infrastructure must be capable of supplying the peak output Q_p, i.e. $H_p=f_{Q_p}\left(D^*_{\{q_t\}}\right)$.

The optimal pipeline design, denoted $D^*_{\{q_t\}}$, minimizes the following annual cost function $SRTCC_{\{q_t\}}\left(D\right)=l.C_D\left(D\right)+\alpha_C.C_3\left(f_{Q_p}\left(D\right)\right)+\frac{1}{365}\sum_{t=1}^{365}C_4\left(f_{q_t}\left(D\right)\right)$. A necessary condition is that the derivative of that cost function with respect to pipeline diameter D be zero:

$$\frac{dSRTCC_{\{q_t\}}}{dD}\left(D^*_{\{q_t\}}\right)=0. \tag{18}$$

As usual, that condition has its economic interpretation. At that optimum, the marginal increase in the pipeline cost is exactly equal to the marginal compression cost reduction.

As a benchmark, it is interesting to compare this optimal pipeline diameter $D^*_{\{q_t\}}$ with those, denoted $D^*_{Q_p}$, that would have been chosen if the firm had had to transport a steady flow Q_p. As the infrastructure must be designed to transport the peak flow Q_p, each of the two following combinations of inputs: $\left(D^*_{\{q_t\}}, f_{Q_p}\left(D^*_{\{q_t\}}\right)\right)$, and $\left(D^*_{Q_p}, f_{Q_p}\left(D^*_{Q_p}\right)\right)$ represents a technology-compatible choice.

> ***Proposition 1:***
> *Assume a gas transmission firm with a seasonal-varying output, costs and engineering production function as described above. For that firm, the long-run optimal combination of inputs $\left(D^*_{\{q_t\}}, f_{Q_p}\left(D^*_{\{q_t\}}\right)\right)$ involves a smaller diameter and a larger horsepower capacity than those that would have been installed to serve a steady daily flow equal to the peak value Q_p.*

A proof of that result is provided in Appendix 2.

This result is rational: a lower load factor creates an incentive to lower the transportation cost by preferring a mix of inputs that includes more compressor horsepower (that generates flexibility) and less pipeline.

3 – Building ahead of demand, an irrational decision?

This section provides a discussion on the optimal investment policies that can be derived from the Chenery-Yépez approach. In his discussion on the economics of the natural gas transportation industry, Yépez (2008, p. 80) suggests that: *"Whatever the planned level of output, the rational firm will select a transportation system whose short-run average total cost is tangent to its LRAC at that capacity"*. It is tempting to confront this rule with the investment decisions actually taken in the gas industry.

3.1 Preliminary remarks

Evidence drawn from the gas industry provides numerous cases of transmission infrastructures that were designed with a significantly oversized diameter. The "Yamal pipeline" – an impressive 56-inch diameter infrastructure that runs from the Yamal peninsula (northern Russia) to Germany across Belarus and Poland – provides an archetypal example. Since its construction in the late 1980s, the gas flow transported on that pipeline has never exceeded 20 Bcm/y, which is a relatively low figure compared to the initial plans (Victor and Victor, 2006). Another case is given by the contemporary Nabucco project, a large gas transmission infrastructure that has been proposed for construction across South-East Europe and Turkey to carry gas from the Caspian region to Austria and other European markets. The design chosen for this project involves a large diameter (56 inches) that looks considerably oversized for the expected flow (8 Bcm/y). In both cases, a cost-minimizing design based on the Chenery-Yépez approach would certainly suggest a smaller diameter/compression horsepower ratio. Why did the teams of skilled and experienced planners who designed those infrastructures prefer alternative solutions based on larger diameters?

This question calls for a closer examination of the model's hypotheses. More precisely, the planned infrastructure is designed to transport a steady flow Q whose value is known and is expected to remain steady during the entire project's life. This assumption allows an analysis of the decision in a static and deterministic framework. In the previous industrial cases, this important condition was not fulfilled. For the "Yamal pipeline", the initial plan was based on the construction of two parallel 56-inch pipelines allowing an export potential of 67 Bcm/y. Up until now, only one of these two pipelines has been built, with a reduced number of compressors from the original plan (Victor and Victor, 2006). As far as Nabucco is concerned, a phased design has also been adopted. It relies on a single large-diameter pipeline with a compression capacity initially adjusted to transport 8 Bcm/y during the first phase. Then, continued additions of compression capacity will ultimately allow an increase in its output to 31 Bcm/y, a large flow that justifies a 56-inch diameter. In both cases, the planner's decision to use an oversized pipeline diameter reflected the perceived massive – but uncertain when the pipeline design was decided – export perspectives to the European gas markets. Stated differently, planners explicitly take into consideration the possibility of there being significant, but still uncertain, expansions in the flows that will be transported during the infrastructure's lifetime.

3.2 Overcapacity, an irrational decision?

The aim of this subsection is to propose a simple extension of the Chenery-Yépez framework to take into consideration the remarks above; hence, to analyze the rationality of such a "building ahead of demand" behavior. This discussion echoes the so-called capacity expansion problem studied in economic theory, a field also pioneered by H.B. Chenery. In his seminal contribution, Chenery (1952) discussed the effect of technology on investment behavior with the goal to illustrate how the simultaneous presence of growing demand and economies of scale may motivate the construction of an oversized equipment. Stated differently, Chenery (1952) proved that *"a building ahead of demand"* decision can be rational. Following that contribution, the optimal degree of excess capacity to be built into a new facility has motivated an admirable stream of literature with a noteworthy extension provided in Manne (1961) that analyzed the case of a random-walk pattern of trends in demand (see Luss, 1982, for a complete survey).

In this subsection, a simple framework is used to analyze whether that "building ahead of demand" behavior can be observed in the gas transmission industry. For expository reasons, a simple discrete time context will be used (extension to continuous version is straightforward). To do so, I focus on the case of a firm that, at date $t=0$, is considering the construction of a single-line transportation infrastructure that is expected to transport a given steady daily flow of gas Q_0 over a predefined planning horizon of Y years. Given the long durability of gas transmission equipments, it is assumed that there will be no equipment replacements during that period.

Compared to the previous models, we now consider the case of a possible future expansion of the output. Hence, there is a known date T, $T \in \{1,...,Y\}$ at the beginning of which the possible output expansion will be decided or not. The decision outcome is still uncertain but is assumed to be restricted to two cases: either a sudden output expansion to a known value Q_1, with $Q_1 > Q_0$; or a *status quo* to Q_0. The probability to observe a rise to Q_1 is denoted p.

If such an additional output was to be decided, the cheapest way to accommodate that additional flow would be the addition of compression horsepower. Hence, the planner's set of decision variables can be restricted to three elements: the pipeline diameter D, the initial compressor horsepower H_0 required to move Q_0, and the compressor horsepower H_1 that could eventually be needed to transport Q_1 from date T to Y.

As above, the annual total cost of the equipment designed to transport a steady daily flow Q_0 during the predefined planning horizon Y is given by $\left[l.C_D(D)+C_H(H_0)\right]$. Besides, planners have to take into consideration a possible extra compression cost in case of an output expansion. It is assumed that the increased horsepower H_1 can be obtained by installing, at the beginning of year T, more compressors in addition to the existing ones at a total cost: $C_3(H_1)-C_3(H_0)$. Clearly, these new compressors will only be used during $Y-T$.

Evaluated at year T, the annual equivalent extra-cost required to install and operate these new compressors during $Y-T$ is denoted $\Delta C(H_0,H_1)$ and is equal to:

$$\Delta C(H_0,H_1)=\alpha_{\Delta H}.\left[C_3(H_1)-C_3(H_0)\right]+C_4(H_1)-C_4(H_0); \quad (19)$$

where $\alpha_{\Delta H}$ is the fixed-cost annual percentage charge dependent on the depreciation and real interest rates to operate those additional compressors during $Y-T$. In fact, $\alpha_{\Delta H}$ is obtained by computing the constant annual outlay stream that has an expected present value equal to that of all future cost outlay over the horizon $Y-T$. In most real cases, we have $\alpha_{\Delta H} > \alpha_{H}$.

A risk-neutral planner is supposed to minimize the expected total annual cost of that infrastructure subject to the usual engineering equations:

$$\begin{aligned} &\underset{D,H_0,H_1}{\text{Min}} \quad \left[l.C_D(D)+C_H(H_0)\right]+p\frac{1}{(1+r)^T}\Delta C(H_0,H_1) \\ &\text{s.t.} \quad F(D,H_0,Q_0)=0 \\ &\qquad\ F(D,H_1,Q_1)=0. \end{aligned} \tag{20}$$

Using the previous modus operandi, this problem can easily be rearranged into a single-variable objective function to be minimized. In fact, the optimal pipeline diameter computed for a probability p is D_p^* and must minimize the expected total annual cost function:

$$\overline{C}(D)= \left[l.C_D(D)+C_H\left(f_{Q_0}(D)\right)\right]+p\frac{\Delta C\left(f_{Q_0}(D),f_{Q_1}(D)\right)}{(1+r)^T}. \tag{21}$$

With the cost specifications presented in Yépez (2008), this is a smooth function that has a unique minimum. Thus, a necessary (and in that case sufficient) condition for a minimum is:

$$\frac{d\overline{C}}{dD}\left(D_p^*\right)=0. \tag{22}$$

That condition has its economic interpretation: at the optimal diameter, the marginal increase in the pipeline cost is exactly equal to the expected marginal reduction in compression cost.

As a benchmark, a planner could find it interesting to confront that outcome with the optimal diameter D_0^* that would have been selected in case of a zero probability for the sudden net increase in output.

Proposition 2:

Assume a gas transmission firm with a probability p for a sudden rise in its output at date T, with costs and engineering production function as described above. For that firm, the long-run optimal combination of equipment to be installed at date $t=0$

involves a pipeline diameter D_p^* that is larger than those, denoted D_0^* that would have been installed by a planner that did not take into consideration this possible future rise in output.

Again, a straightforward proof of that result is provided in Appendix 2.

This result has important implications for the design of appropriate regulatory policies. In many countries, the level of prices charged by gas transmission firms is subject to public control. In many cases, a rate of return regulation is implemented. This form of regulation sees costs as exogenous and observable and forms prices on the basis of observed costs and the appropriate rate of return on capital. One of the principal criticisms that has arisen for this kind of regulation is based on the so-called Averch-Johnson effect. According to Averch and Johnson (1962), the profit-seeking behavior of the regulated firm subject to rate of return regulation induces a distortion in the input choice: the optimal choice of that firm is not the cost-minimizing one. More precisely, the capital/labour ratio chosen by the firm subject to rate of return regulation is greater than the cost-minimizing capital/labor ratio for the given level of output. Obviously, such a statement calls for a condemnation of the tendency of regulated firms to engage in excessive amounts of durable capital accumulation to expand the volume of their profits. It also suggests some reforms, such as a regulatory control over the input choice, or the implementation of more sophisticated regulatory approaches (Laffont and Tirole, 1993). The case analyzed above suggests a different explanation for the firm's preference for a capital intensive technology. Here, the firm's decision to choose an input mix that includes a larger diameter/horsepower ratio – i.e. a greater capital/energy ratio – is completely independent of the regulatory environment, such a choice is entirely motivated by an anticipation of possible future output expansion.

Conclusions

Throughout this article the cost functions for the natural gas transmission industry presented in Chenery (1949) and Yépez (2008) are discussed in the light of some key features of that industry. A commented review of the Chenery-Yépez methodology has enabled the derivation of some interesting insights, such as an empirical quantification of the scale economies encountered in that industry and a straightforward single-variable reformulation of the long-run problem. Two notable extensions have also been provided to deal (a) with the case of a seasonal varying output, and (b) with those of an uncertain future output expansion. Besides, the discussions have also highlighted a couple of important implications for the design of appropriate regulatory policies in that industry.

The Chenery-Yépez method has great merit: it offers a simple engineering-based approach to determine the analytical cost functions encountered in the natural gas transmission industry. For this reason, it is worth being considered as a valuable tool to get a better understanding of the gas pipeline economics. Moreover, it provides a useful complement to the purely statistical approach in the determination of production, cost, and factor demand relationships. A tribute must thus be paid to the late H.B. Chenery for his inspiring work.

Of course, the gas transmission infrastructure studied in this paper – a single compressor and a pipeline – is a simple one. In many countries, the development of the natural gas industry came with the construction of a complex transmission network. The optimal designs of these infrastructures have generated an affluent operations research literature and many numerical models have been proposed. Yet, very few analytical results have been obtained for these general cases. As an exception, a recent contribution (André and Bonnans, 2010) provides some analytical recommendations for the optimal design of a long-distance pipeline that has several compressor stations installed along the pipe. This is clearly a more complex case than Chenery (1949) as it involves the determination of the pipe-diameters, the size, the number and the spacing of the compressor stations. Technologically, a variation in station spacing can somehow be viewed as an alternative for varying the horsepower per station. The expansion of the previous results to this more complex setting constitutes an attractive research challenge.

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APPENDIX 1

All the numerical simulations presented in this paper are entirely based on the following assumptions chosen for a hypothetical 100-mile long project (l = 100 miles). The technical specifications for the gas are exactly those used in Yépez (2008):

T_b base temperature	520°R
T mean flowing temperature	535°R
P_b base pressure	14.73 psia
P_1 initial pressure in the pipe	1070 psia
P_2 terminal pressure in the pipe	838 psia
G gas specific gravity for the gas in the region	0.62
Z compressibility factor	0.8835
β dimensionless constant	0.22178

Hence, the dimensionless constants have the following values: c_0 = 0.742 and c_1 = 183.2 (Yépez, 2008).

Publicly available information on cost data for that industry is rather scarce. As far as costs are concerned, the data come from Yépez (2008) and seem to be the result of a statistical investigation. Despite the absence of some of the classical attributes of a typical empirical study (e.g.: a description of the data, a discussion on the regression results: t-statistics, R^2...), we can reasonably consider that Yépez (2008) constitutes a reliable and publicly available source that should be preferred to many a lesser man's studies.

The annual cost parameters (in US Dollars) are thus those presented in Yépez (2008):

$$\alpha_L.C_1(D,\tau) = 7144.59D^{0.881}\tau^{0.559};$$

$$C_2(D) = 317.61D^{0.809};$$

$$\alpha_C.C_3(H) = 1256.33H^{0.9016};$$

$$C_4(H) = 6145.177H^{0.4523};$$

Moreover, the pipe thickness τ is assumed to be a linear function of D the inside diameter: $\tau = \dfrac{D}{110}$. This formulation has been suggested by concerns about pipe stability (cf. Ruan et al., 2009, p. 3044). Hence, the annual cost of the line per mile is $\alpha_L.C_1(D) = \dfrac{7144.59}{110^{0.559}} D^{0.881+0.559}$.

APPENDIX 2

***Proof of Lemma 1**:*

For a given flow of gas to be transported Q, it is clear that $SRTC_Q\left(D_{SR}^*\right) \geq LRTC\left(D_{LR}^*, H_{LR}^*\right)$ (otherwise, there would be an obvious contradiction with $\left(D_{LR}^*, H_{LR}^*\right)$ being the unique optimal solution to the programme (2) as the combination $\left(D_{SR}^*, f_Q\left(D_{SR}^*\right)\right)$ would provide a lower cost). We now have to prove that $SRTC_Q\left(D_{SR}^*\right) = LRTC\left(D_{LR}^*, H_{LR}^*\right)$. As a textbook example of a reductio ad absurdum, we assume that $SRTC_Q\left(D_{SR}^*\right) > LRTC\left(D_{LR}^*, H_{LR}^*\right)$ and have a closer look at the solution $\left(D_{LR}^*, H_{LR}^*\right)$. As that solution imperatively satisfies the engineering equation (1), we have a relation between D_{LR}^* and H_{LR}^*: $H_{LR}^* = f_Q\left(D_{LR}^*,\right)$. Hence, $SRTC_Q\left(D_{SR}^*\right) > LRTC\left(D_{LR}^*, H_{LR}^*\right)$ corresponds to $SRTC_Q\left(D_{SR}^*\right) > l.C_D\left(D_{LR}^*\right) + C_H\left(f_Q\left(D_{LR}^*\right)\right)$. Given that the right-hand side of that equation is nothing but $SRTC_Q\left(D_{LR}^*\right)$, we have an obvious contradiction with D_{SR}^* being the unique diameter that minimizes the Short-Run Total Cost to transport Q. *Q.E.D.*

We shall need one technical lemma before we can prove more useful results. Remember that: $\beta \in (0,1)$ is a technological parameter that intervenes in (2), and $g_D(Q) = f_Q(D)$ is the horsepower demand function defined in (9). Hereafter, we denote: $h_D : Q \mapsto f_Q^{'}(D)$ the marginal impact of the pipe diameter on the horsepower required to move the gas; $C(H)$ a compressor cost function; and $k_D : Q \mapsto h_D(Q).C^{'}\left(g_D(Q)\right)$ a smooth function that gives the marginal impact of the pipe diameter on the compression cost to transport Q on an infrastructure with a pipe diameter equal to D.

> **Lemma 2 (technical):** *Assume the following specification for the compressor cost function: $C : H \mapsto a.H^b$ with $0 < b < 1$, there exists a non-empty interval $\left(\underline{b}_\beta, \overline{b}_\beta\right)$ that depends on β, so that: for all $b \in \left(\underline{b}_\beta, \overline{b}_\beta\right)$, the gradient $\dfrac{dk_D}{dQ}(Q)$ is strictly negative for all $Q > 0$.*

Proof of Lemma 2:

To simplify the notations, we denote $\theta = l.\left(c_0\right)^{-2}$. The sketch of the reasoning is as follows. After rearranging, this gradient can be re-written as: $k^{'}_D(Q) = A(Q).B(Q).E(Q)$, where:

$$A(Q)=-\frac{8}{3}\cdot\frac{c_1^2.Q^3.\beta.\theta.C'\left(g_D(Q)\right)}{D.g_D(Q)}\cdot\left(\theta.Q^2+D^{\frac{16}{3}}\right)^{-2};\qquad B(Q)=\left(\frac{\theta.Q^2}{D^{16/3}}+1\right)^{\frac{\beta}{2}}\qquad and$$

$$E(Q)=\left[b.\theta.\left(B(Q)-1\right)+\theta.\beta.\left(B(Q).b-1\right)\right].Q^2+D^{\frac{16}{3}}.\left[b.\left(B(Q)-1\right)+2.\left(B(Q)-1\right)\right].$$

We remark that: $A(Q)<0$ *and* $B(Q)>1$ *for all* $Q>0$. *As* $\left(B(Q).b-1\right)$ *can take negative values, we cannot affirm that* $E(Q)$ *is strictly positive for all* Q. *Nevertheless, we remark that* E *is a smooth function that verifies* $E(0)=0$ *and* $E'(0)=0$. *Hereafter, we are going to propose some conditions on* b *that are sufficient to insure the strict convexity of* E *and focus* E''. *Interestingly, if* $b\geq\underline{\mu_1}$ *where* $\underline{\mu_1}=(1-\beta).(3\beta+4)^{-1}$, *the fact that* $B(Q)\geq 1$ *is sufficient to obtain:*

$$E''(Q)\geq F(Q).\left(\theta.Q^2+D^{\frac{16}{3}}\right)^{-2},\qquad \forall Q, \tag{23}$$

where $F(Q)$ *is a "well-behaved" polynomial of degree 4: a quadratic polynomial of the variable* Q^2. *Looking at* $F(Q)$, *we can notice: (i) that the coefficient of* Q^4 *is strictly positive iff* $b>\underline{\mu_2}$ *where* $\underline{\mu_2}=2.\left(\beta^2+4\beta+5\right)^{-1}$ *and (ii) that the constant coefficient is strictly positive. So, if* $b>\underline{\mu_2}$ *and the discriminant of this quadratic polynomial of* Q^2 *is negative, then* $F(Q)$ *will have no real root and it will take only strictly positive values. Interestingly, this discriminant is directly proportional to* $\Phi(b)=\left(1+12\beta+6\beta^2\right)b^2-\left(18+10\beta-6\beta^2\right)b+9-6\beta+\beta^2$ *which is strictly negative iif: (i) the polynomial* $\Phi(b)$ *has two real roots* $\underline{\mu_3}$ *and* $\overline{\mu_4}$ *with* $\underline{\mu_3}<\overline{\mu_4}$, *and (ii)* $b\in\left(\underline{\mu_3},\overline{\mu_4}\right)$.

Hereafter, we denote $\underline{b}_\beta=Max\left(\underline{\mu_1},\underline{\mu_2},\underline{\mu_3}\right)$ *and* $\overline{b}_\beta=Min\left(\overline{\mu_4},1\right)$ *and we can notice that: for all* $\beta\in(0,1)$, *the interval* $\left(\underline{b}_\beta,\overline{b}_\beta\right)$ *is non-empty. So, if* $b\in\left(\underline{b}_\beta,\overline{b}_\beta\right)$, *all the conditions above are simultaneously satisfied and (23) insures that* E *is a strictly convex function. As* $E'(0)=0$, *it is now clear that* $E'(Q)>0$ *for all* $Q>0$. *As* E *is a strictly increasing function for all* $Q>0$ *that verifies* $E(0)=0$, *the function* E *only takes strictly positive values for all* $Q>0$. *As* $A(Q)<0$ *and* $B(Q)>1$ *for all* $Q>0$, *the gradient* $k'_D(Q)=A(Q).B(Q).E(Q)$ *is thus strictly negative for all* $Q>0$. *Q.E.D.*

Proof of Proposition 1:

The short-run total cost to transport the daily flows $\{q_t\}$ on a transmission infrastructure designed to transport a peak output Q_p is given by:

$$SRTCC_{\{q_t\}}(D) = SRTC_{Q_p}(D) - \frac{1}{365}\sum_{\substack{t=1 \\ q_t<Q_p}}^{365}\left[C_4\left(f_{Q_p}(D)\right) - C_4\left(f_{q_t}(D)\right)\right],$$

where $SRTC_{Q_p}(D)$ is the function previously described that gives the short-run total cost to transport a steady flow Q_p and $SRTCC_{\{q_t\}}(D)$ is the short-run total contraction cost. The derivative of that second function w.r.t. D is equal to:

$$\frac{dSRTCC_{\{q_t\}}}{dD}(D) = \frac{dSRTC_{Q_p}}{dD}(D) - \frac{1}{365}\sum_{\substack{t=1 \\ q_t<Q_p}}^{365}\frac{d}{dD}\left[C_4\left(f_{Q_p}(D)\right) - C_4\left(f_{q_t}(D)\right)\right].$$

As the diameter $D^*_{Q_p}$ satisfies condition (13), we have:

$$\frac{dSRTCC_{\{q_t\}}}{dD}\left(D^*_{Q_p}\right) = \frac{1}{365}\sum_{\substack{t=1 \\ q_t<Q_p}}^{365}\left[f'_{q_t}\left(D^*_{Q_p}\right)\times C'_4\left(f_{q_t}\left(D^*_{Q_p}\right)\right) - f'_{Q_p}\left(D^*_{Q_p}\right)\times C'_4\left(f_{Q_p}\left(D^*_{Q_p}\right)\right)\right].$$

Hereafter, we are going to show that this value is strictly positive. We denote: $h_D : Q \mapsto f'_Q(D)$ the marginal impact of the pipe diameter D on the compression horsepower required to transport Q on a given infrastructure, and $k_D(Q) = h_D(Q).C'_4\left(g_D(Q)\right)$ the marginal impact of the pipe diameter on the compression cost incurred to transport Q on that infrastructure. Two cases must be distinguished depending on the assumed specification for the operation cost of the compressor:

- <u>*Case #1:*</u> *Chenery's linear specification is assumed: $C_4(H) = C'_4.H$ with $C'_4 > 0$. Here, $k_D(Q)$ is directly proportional to $h_D(Q) = \frac{-8.\beta.c_1.l}{3.c_0^{\,2}.D^{19/3}}.Q^3.\left(\frac{l}{c_0^{\,2}.D^{16/3}}.Q^2+1\right)^{\frac{\beta}{2}-1}$. As $\beta \in (0,1)$, the gradient of h_D w.r.t. Q is strictly negative for all $Q>0$ and so is $k_D{}'(Q)$.*

- *Case #2:* *Yépez's concave specification is assumed:* $C_4\left(H\right)=d.H^{\psi}$ *with* $d>0$ *and* $\psi=0.4523$. *In that case, we can check whether the conditions stated in Lemma 2 are verified or not. With the numerical parameters provided in Yépez (2008),* $\beta=0.22178$ *and* $\left(\underline{b}_{\beta},\overline{b}_{\beta}\right)\approx\left(0.423\ ,\ 1\right)$. *As* $\psi\in\left(\underline{b}_{\beta},\overline{b}_{\beta}\right)$, *the gradient of* k_D *w.r.t.* Q *is strictly negative for all* $Q>0$ *(Lemma 2).*

So, in both cases: the smooth function k_D *is strictly decreasing. As a strictly less than one load factor is assumed, there is at least a day* t *that verifies* $q_t<Q_p$ *and thus:* $f_{q_t}^{'}\left(D\right)\times C_4^{'}\left(f_{q_t}\left(D\right)\right)-f_{Q_p}^{'}\left(D\right)\times C_4^{'}\left(f_{Q_p}\left(D\right)\right)>0$. *Gathering these inequalities across all the days* t *with* $q_t<Q_p$, *and evaluating them for* $D=D_{Q_p}^{*}$ *insures that* $\dfrac{dSRTCC_{\{q_t\}}}{dD}\left(D_{Q_p}^{*}\right)>0$.

Given that: (i) the short-run total contraction cost is locally strictly increasing with D *in the neighbourhood of the particular diameter* $D_{Q_p}^{*}$, *and (ii)* $SRTCC_{\{q_t\}}$ *is a differentiable function with a unique minimum, it indicates that the optimal diameter* $D_{\{q_t\}}^{*}$ *is on the left of* $D_{Q_p}^{*}$ *which means that* $D_{\{q_t\}}^{*}<D_{Q_p}^{*}$. *Thus, a gas transmission firm that serves a fluctuating demand prefers to choose a mix of inputs* $\left(D_{\{q_t\}}^{*},f_{Q_p}\left(D_{\{q_t\}}^{*}\right)\right)$ *based on a smaller diameter and a larger compression capacity than those chosen in case of a steady demand equal to* Q_p, *i.e.:* $\left(D_{Q_p}^{*},f_{Q_p}\left(D_{Q_p}^{*}\right)\right)$. *Q.E.D.*

Proof of Proposition 2*:*

As in the Proof of Proposition 1, the sketch of the proof relies on an evaluation of the sign of the derivative of the expected total annual cost function with respect to pipeline diameter D *evaluated for the particular diameter* D_0^{*}. *As* $\overline{C}$ *is a smooth function with a unique minimum, this information allows us to conclude on the relative size of* D_0^{*} *and* D_p^{*} *because a negative (conversely positive) value clearly suggests that* $D_0^{*}<D_p^{*}$ *(conversely* $D_0^{*}>D_p^{*}$*).*

D_0^{*} *satisfies condition (22) in the particular case of* $p=0$. *In case of* $p>0$, *the derivative of* $\overline{C}$ *with respect to* D *evaluated at* $D=D_0^{*}$ *is equal to:*

$$\frac{d\overline{C}}{dD}\left(D_0^*\right) = p.\frac{1}{\left(1+r\right)^T}.\frac{d}{dD}\Delta C\left(f_{Q_0}\left(D_0^*\right), f_{Q_1}\left(D_0^*\right)\right), \tag{24}$$

where $\Delta C\left(f_{Q_0}(D), f_{Q_1}(D)\right) = \alpha_{\Delta H}.\left[C_3\left(f_{Q_1}(D)\right) - C_3\left(f_{Q_0}(D)\right)\right] + C_4\left(f_{Q_1}(D)\right) - C_4\left(f_{Q_0}(D)\right)$.

Let's start with the operation cost of the compressor C_4. *It has been proven (cf. Proposition 1) that the marginal impact of the pipe diameter* D *on the operation cost of the compressor - i.e. the function* $k_D(Q) = h_D(Q).C_4^{'}\left(g_D(Q)\right)$ *- is strictly decreasing with* Q. *Hence, we have:*

$$f_{Q_1}^{'}(D).C_4^{'}\left(f_{Q_1}(D)\right) - f_{Q_0}^{'}(D).C_4^{'}\left(f_{Q_0}(D)\right) < 0, \qquad \text{for all } Q_1 > Q_0. \tag{25}$$

A similar line of reasoning can also be applied with the capital cost of the compressor C_3. *Renaming* $k_D : Q \mapsto f_Q^{'}(D).C_3^{'}\left(f_Q(D)\right)$, *two cases can also be considered depending on the specification assumed for the capital cost of the compressor. If (Case #1) Chenery's linear specification is assumed - i.e.* $C_3(H) = C_3^{'}.H$ *with* $C_3^{'} > 0$, *we know (cf. Proposition 1) that* $k_D^{'}(Q) = C_3^{'}.h_D^{'}(Q)$ *is strictly negative for all* $Q > 0$. *If (Case #2) Yépez's specification is assumed - i.e.* $C_3(H) = c.H^{\varepsilon}$ *with* $c > 0$ *and* $\varepsilon = 0.9016$, *the value of the compression parameter used in Yépez (2008) - i.e.* $\beta = 0.22178$ *- is compatible with* $\varepsilon \in \left(\underline{b}_\beta, \overline{b}_\beta\right)$ *as defined in Lemma 2. Hence, the gradient of* k_D *w.r.t.* Q *is strictly negative for all* $Q > 0$. *So, in both cases,* $k_D : Q \mapsto h_D(Q).C_3^{'}\left(g_D(Q)\right)$ *is strictly decreasing i.e.:*

$$f_{Q_1}^{'}(D).C_3^{'}\left(f_{Q_1}(D)\right) - f_{Q_0}^{'}(D).C_3^{'}\left(f_{Q_0}(D)\right) < 0, \qquad \text{for all } Q_1 > Q_0. \tag{26}$$

Combining (25) and (26), the derivative of the incremental compression cost $\Delta C\left(f_{Q_0}(D), f_{Q_1}(D)\right)$ *w.r.t.* D *is strictly negative for all* $Q_1 > Q_0$. *Thus,* $\frac{d\overline{C}}{dD}\left(D_0^*\right) < 0$ *for all* $Q_1 > Q_0$. *Q.E.D.*