Seismic response of Roman concrete arches and vaults

Martin Williams, University of Oxford

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Structures and dynamics at Oxford

• Based within a unified Engineering Science department of about 100 academics

• Key personnel:
  – Martin Williams, Tony Blakeborough, Manolis Chatzis, Zhong You (Structures)
  – Byron Byrne, Ross McAdam (Geotechnics)
  – Tom Adcock (Fluids)

• Topics:
  – real-time hybrid test methods
  – passive energy dissipation systems in earthquake engineering
  – human-structure interaction in grandstands and footbridges
  – passive control of suspension bridge flutter
  – non-linear system identification
  – rocking mechanics
  – deployable and origami structures
  – dynamics of offshore renewable energy systems
Contents

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• Case study of the Basilica of Maxentius
  – history
  – site surveys
  – limit analyses
• Laboratory testing of mortar arches
  – tests on uncracked arches
  – tests on arches with pre-existing cracks
  – analysis
• Conclusion
Introduction – Roman concrete

- Alternating layers of pozzolanic mortar and layers of fist-sized aggregate (*caementa*: broken bricks, stones) – “mortared rubble”
- Often cast between brick facings, and with occasional layers of flat bricks (*bipedales*)
- Properties broadly similar to modern concretes

Lancaster (2009) Concrete vaulted construction in Imperial Rome
Introduction – vaulted Roman structures

• A structural form dating from 1st century BC

• Principal forms:
  – barrel vaults
  – cross vaults
  – domes
The Basilica of Maxentius

- Built 307-313 AD on the Via Sacra, just north of the Colosseum
- The largest vaulted space in antiquity – overall plan dimensions 100 x 80 m
- Central nave – three cross-vaults each 25 x 21 m in plan
- Side naves – barrel vaults of 23.5 m diameter
- Built on sloping ground (9m height drop from NE to SW corner), and on remains of earlier structures
The Basilica of Maxentius

• Partial collapse at unknown date in early Middle Ages “widely believed to have been caused by an earthquake”
• Cross-vaults and one range of barrel vaults were lost, leaving only the barrel vaults of the northern nave
• Subsequently, many changes of ground level both within and outside the remaining structure
• Changes from 15th century onwards can be traced through contemporary engravings
Images of the remains

- Duperac (1575)
Images of the remains

- Sylvestre (1650)
Images of the remains

- Vasi (1752)
Images of the remains

- Rosini (1839)
Images of the remains

• Present day:
Aims of surveys and analyses of remains

• Establish accurate geometry
• Quantify structural deformations
• Reconstruct geometry of the lost cross vaults
• Establish state of foundations

and hence...

• Assess stability of original structure, and likelihood of collapse due to earthquake
• Assess stability of remains
Foundation survey

• Shows signs of significant lateral loads in the past
  – diagonal cracking of walls
  – leaning of walls
  – horizontal slip at bonding courses
Point cloud survey
Stability of the remaining barrel vaults

- Distortion of barrel vaults close to outside wall
- Diagnosed as a construction modification to avoid overlapping with windows in the wall
Thrust line analysis

- Concrete density varied from 13.5 kN/m³ at top to 22 kN/m³ near base
- Plots show thrust line corresponding to minimum horizontal reaction
- Variation from circular intrados shape increases minimum thrust from 300 to 335 kN
- ...but structure remains stable
Deformation of west wall

- Outward lean of 0.8° at west end, and distortion of end barrel vault
- Consistent with 4-hinge mechanism
- Close to stability limit at most damaged cross-section
Stability through the ages

• Original structure: 313 AD

• After collapse of central nave, raised ground levels: pre-1500
Stability through the ages

- First excavation: c. 1820

- Second excavation: c. 2000
Reconstruction of cross-vault geometry

- Accurate scaled reconstructions of elements (springings, buttresses, fragments) achieved using digital photogrammetry
- These can then be used to infer overall vault geometry
Stability analysis by slicing technique

- Analysis by slicing technique shows the vault to be stable under self-weight loads
- Imposes lateral thrust of 2.4 MN on the vulnerable west wall
- But this is only one of many possible equilibrium solutions
Conclusion (1)

• Survey has provided most accurate available data on:
  – the current geometry and condition of the remaining structure
  – the likely dimensions of the collapsed part
• Evidence of significant lateral loading in the past
• But still no compelling evidence of cause of collapse
• Original structure shown to be stable under gravity loads
• Remaining structure has only small factor of safety against collapse, due to changes in ground level
Laboratory testing of mortar arches

- Loosely inspired by the Basilica of Maxentius
- Shaking table tests of semi-circular mortar arches
- Aims:
  - understand differences in behaviour between continuous and voussoir arches
  - estimate lateral load capacity of continuous arches
  - investigate significance of pre-cracking of continuous arches under gravity loads
Experiment set-up

• Tests on small-scale arches with thickness:radius ratios of 0.13, 0.14, 0.15
• Pulse-type base motions
• Arch motion measured by PIV
Typical four-hinge collapse

- Cracks have formed in a previous load cycle
- Mechanism rocks and then collapses
- Once formed, hinge locations are fixed
Four-link chain analogy

- Arch is free to rotate at feet (two hinges)
- Two further hinges required to form mechanism
- Arch then free to rotate as a four-bar chain
Comparison with voussoir arch

- Hinges can form at many possible locations
- Travelling hinges possible
- When motion reverses direction, mirror-image hinges form
Hinge locations

- For continuous arch, hinge locations are far more symmetrical than for the voussoir arch
- Two hinges always form at feet – only points of zero tensile strength
- Others at about one-third span points
Can we predict hinge locations?

- Using thrust line analysis...
- With self-weight and lateral acceleration of 0.5g, thrust line and tensile stress envelope are as shown
- Implies highly asymmetric hinge locations
- But thrust line was found based on an assumption that hinges B and C form simultaneously – in reality they don’t
- Without this (or some similar) assumption, problem is indeterminate
Can we predict hinge locations?

- Using elastic FE analysis...
- Again not a good prediction
Failure of a pre-cracked arch

- Single crack at crown generated by lateral spreading of footings
Hinge locations

- Hinge forms approximately midway between crown crack and support
- Lateral acceleration to cause failure is not significantly reduced by pre-cracking
Can we predict final hinge location?

- Yes, using simple quasi-static elastic analysis
Expanding analysis to consider any hinge position

- Tensile stress distributions for different locations of the third hinge:

\[ \frac{t}{R} = 0.14 \]
\[ \frac{t}{R} = 0.15 \]

- For different horizontal accelerations:
  - \( a_H = 5 \text{ m/s}^2 \)
  - \( a_H = 10 \text{ m/s}^2 \)
Returning to initially uncracked arches

- Applying same analysis to formation of final crack:
Conclusion (2)

- Continuous arches/vaults with modest tensile strength develop different hinge patterns to voussoir arches
- Near-simultaneous formation of two hinges – location of first hinge cannot be predicted by quasi-static methods
- Location of final hinge can be predicted using elastic analysis
- Pre-existing cracks have dramatic effect on collapse mechanism
Thank you!