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The tuned mass-damper-inerter (TMDI) for passive vibration control of multi-storey building structures subject to earthquake and wind excitations

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EPSRC

Pioneering research and skills

ideas FACTORY

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ACKNOWLEDGEMENTS

- Prof. Nick Karcanias

2011:
Do you know about what the inerter is?
Can we use it for passive vibration control of buildings?

- Marian Laurentiu

City University London studentship (2012-2015)

PhD thesis (viva 12/2016):
"The tuned mass damper inerter for vibration suppression and energy harvesting in dynamically excited structural systems"

- EPSRC

Big Pitch- Bright Ideas award (10/2014)

Multi-objective performance-based design of tall buildings using energy harvesting enabled tuned mass-damper-inerter (TMDI) devices

EPSRC Pioneering research and skills

ideas FACTORY

Grant Ref: EP/M017621/1

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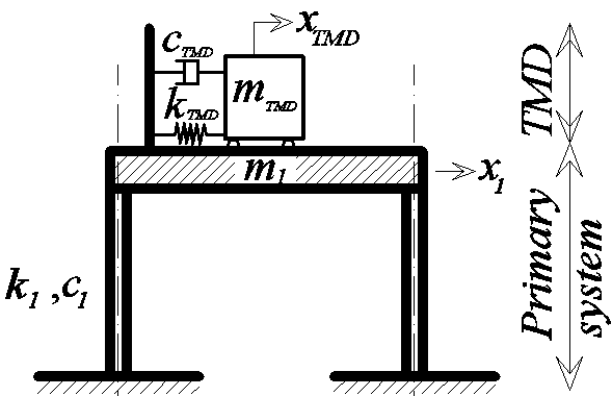
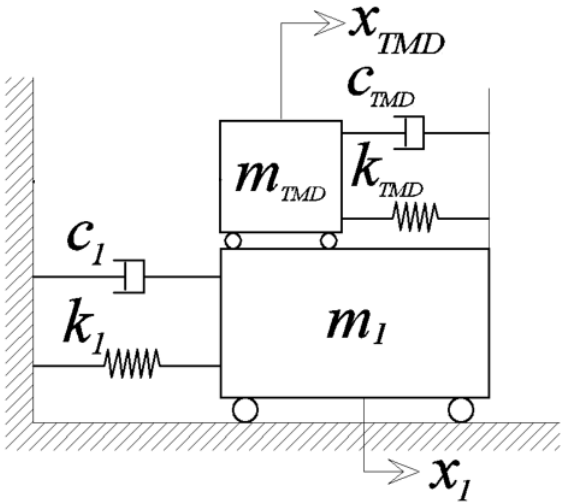
Presentation Outline

- **Introduction/Motivation**
  - The classical tuned mass-damper (TMD)
  - Motivation for the tuned mass-damper-inerter (TMDI)
- **TMDI for single-degree-of-freedom (SDOF) primary structures**
  - Optimum design formulae
  - Vibration suppression performance and weight reduction
- **TMDI for multi-storey seismically excited structures**
  - Eurocode 8 compatible seismic design and assessment of TMDI equipped building structures
  - Reliability-based optimum design of TMDI equipped building structures accounting for higher modes of vibration
- **TMDI for wind excited tall buildings**
  - Vortex shedding and occupants comfort
  - Parametric analysis for a benchmark wind excited TMDI equipped 74-storey building
- **Concluding remarks**

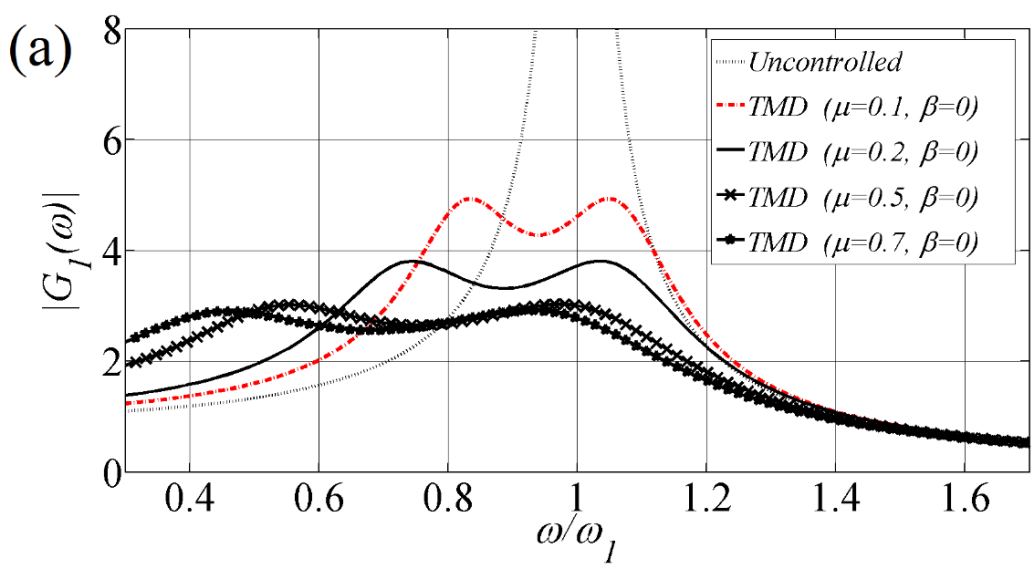
The classical tuned mass damper (TMD)

For primary structures modelled as single-degree-of-freedom (SDOF) systems



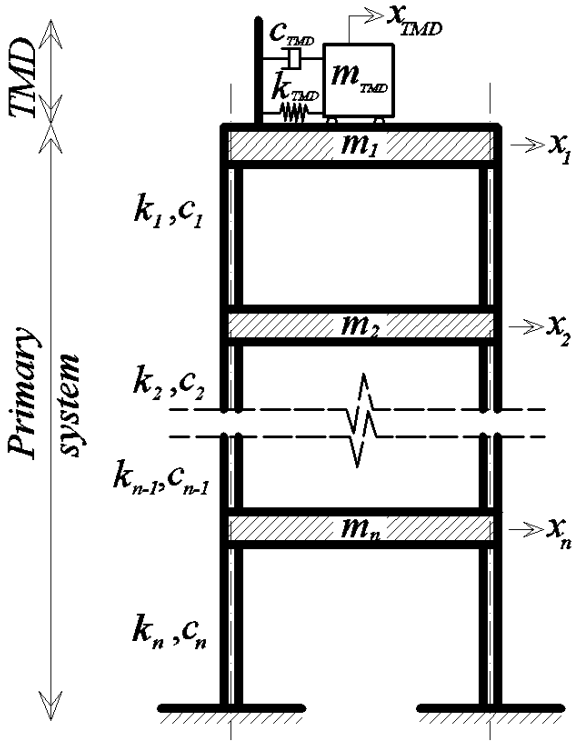


- Additional mass m_{TMD} attached to the primary structure via a linear spring k_{TMD} and dashpot c_{TMD} in parallel.
- Given a primary structure and attached mass, find k_{TMD} and c_{TMD} such that the response of the primary structure is minimised.
- The larger the attached mass, the better level of vibration suppression is achieved in terms of peak response along a wider frequency band.

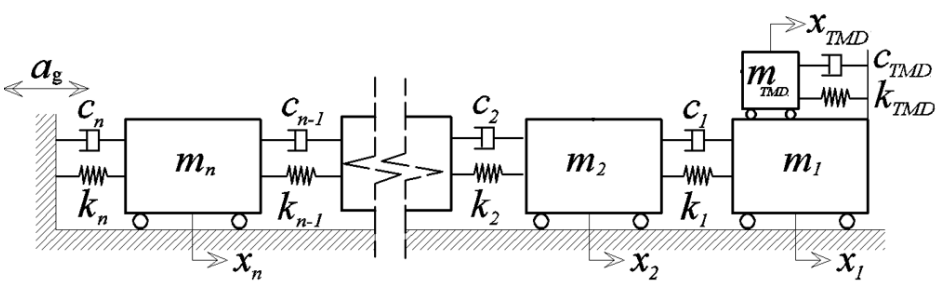


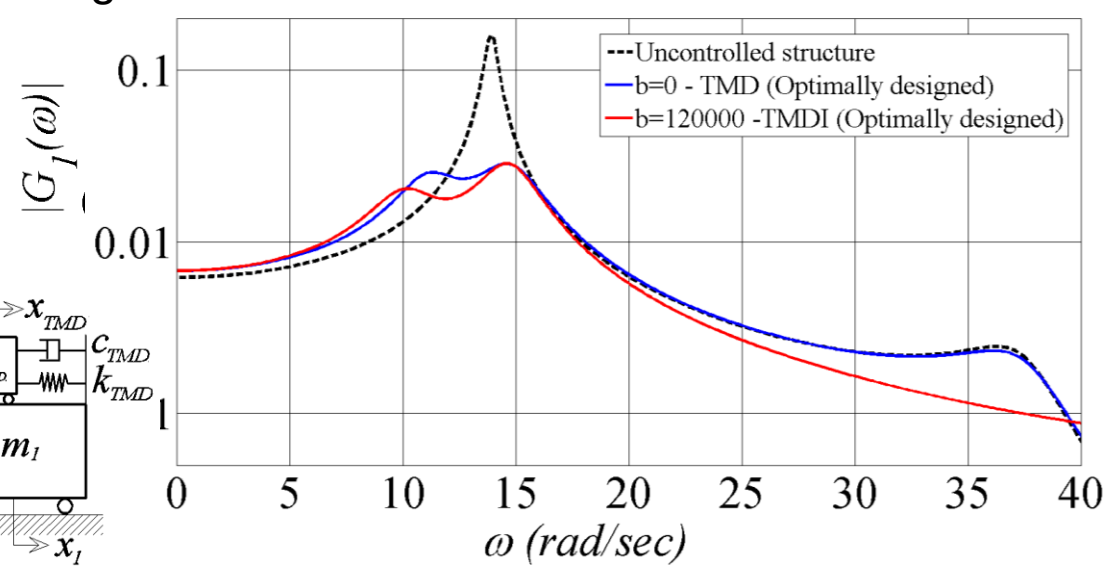
The classical tuned mass damper (TMD)

For primary structures modelled as multi-degree-of-freedom (MDOF) systems



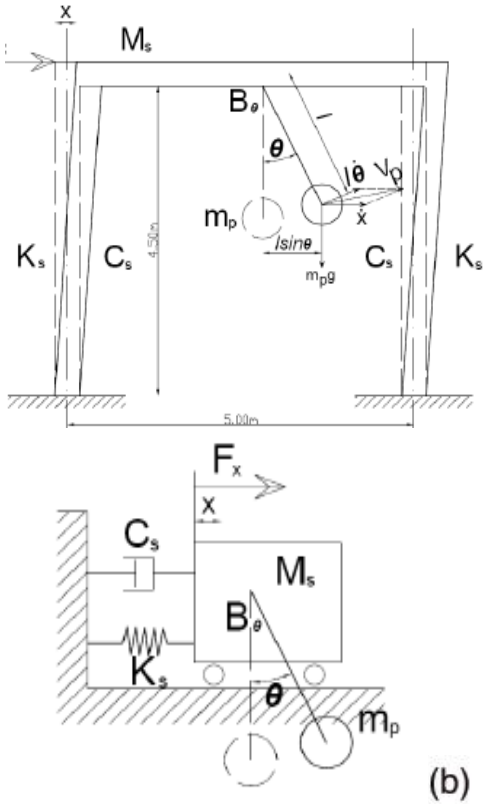
- The m_{TMD} mass is attached to the "lead" mass via a linear spring k_{TMD} and dashpot c_{TMD} in parallel.
- Given a primary structure and attached mass, find k_{TMD} and c_{TMD} to control the first (fundamental) mode shape.
- Wind induced vibrations: m_{TMD} 0.2% - 2% of the total building mass
- Earthquake-induced vibrations: m_{TMD} >>2% of the total building mass

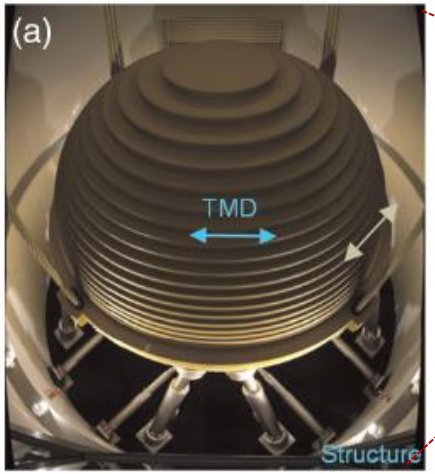




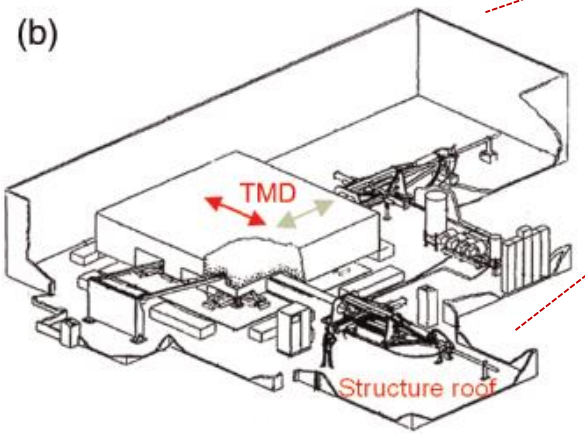
TMD applications in buildings

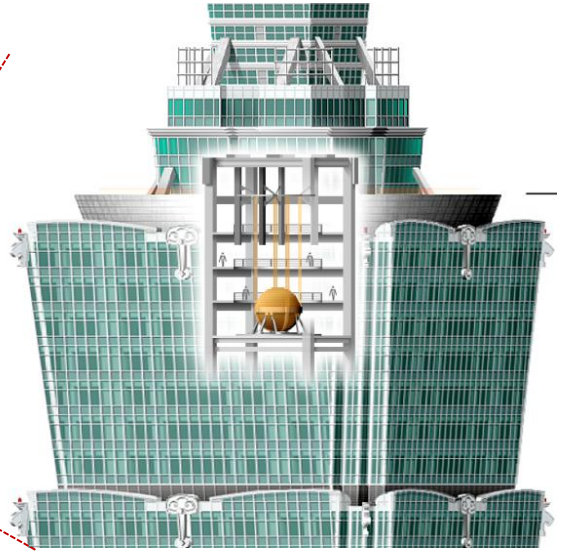
Suppression of wind induced vibrations



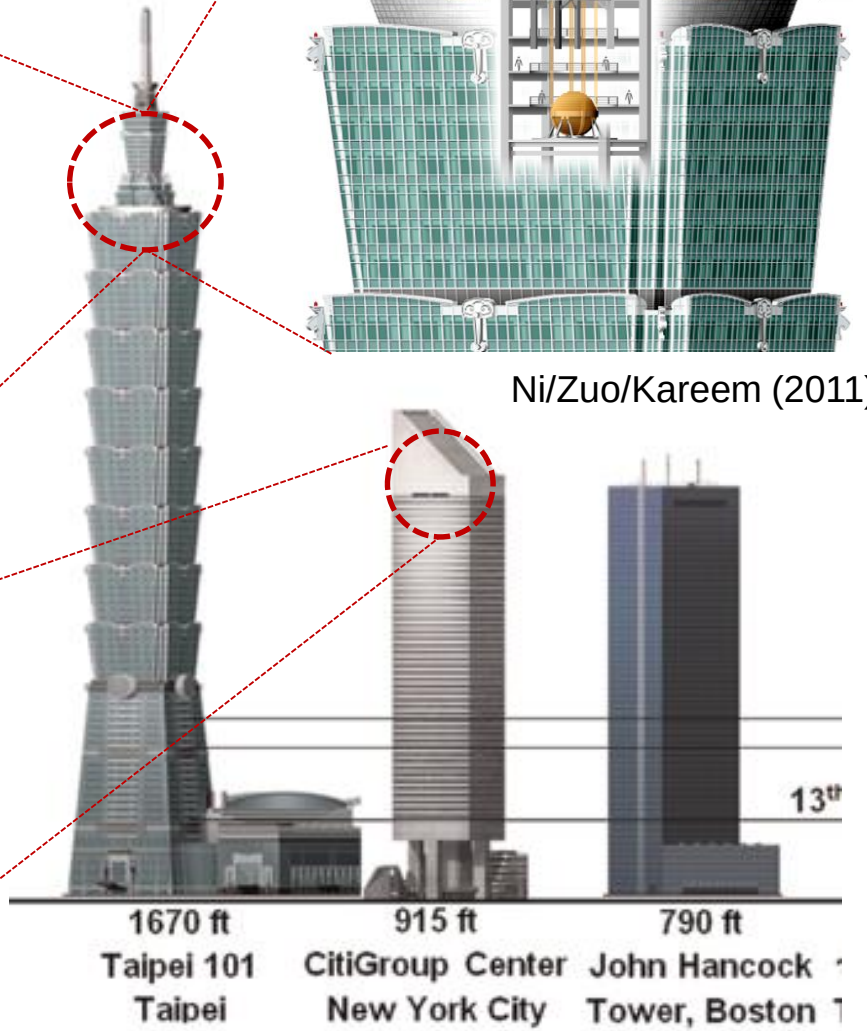


Zuo/Tang, *JIMSS* (2013)





Ni/Zuo/Kareem (2011)



TMD applications in buildings

Edificio Park Araucano, Santiago, Chile

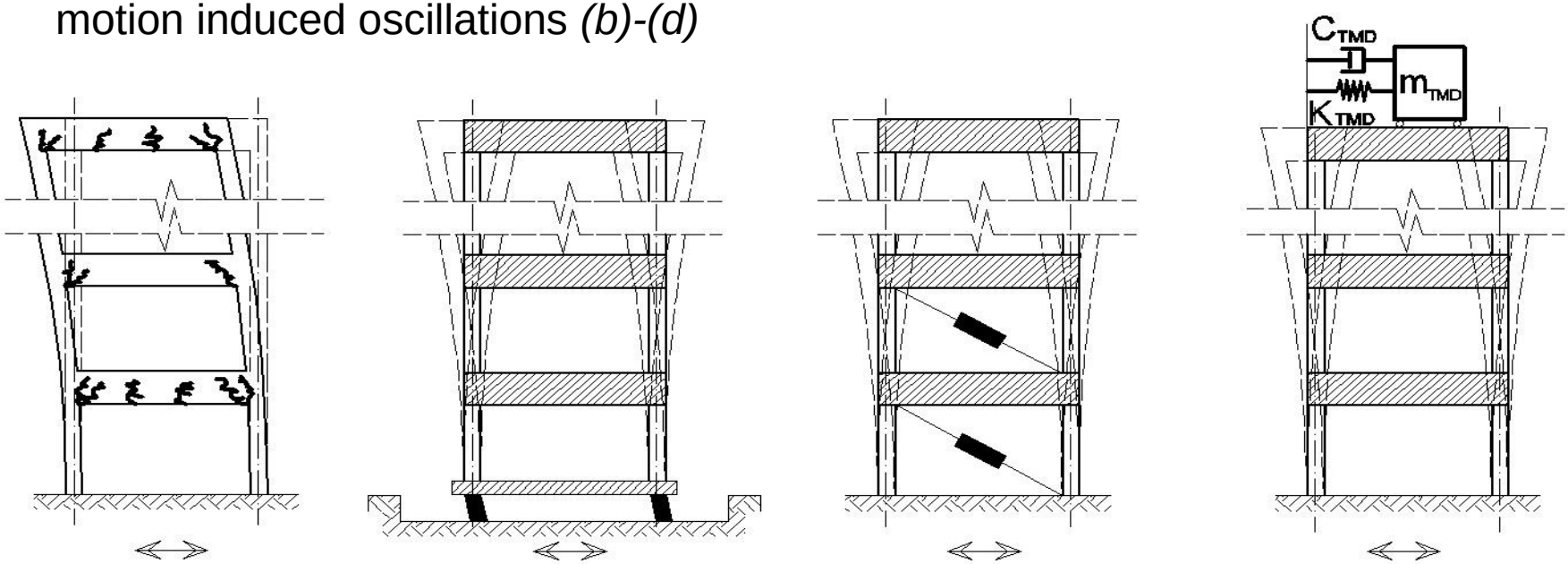
Suppression of earthquake induced vibrations

http://sirve.cl/archivos/proyectos/diseno-de-sistema-de-proteccion-sismica-edificio-parque-araucano

Motivation for the tuned mass-damper-inerter (TMDI)

➢ Protection of buildings for earthquakes

- Conventional design philosophy: allows for inelastic deformations *(a)*
- Passive vibration control: incorporates different devices to reduce the ground motion induced oscillations *(b)-(d)*



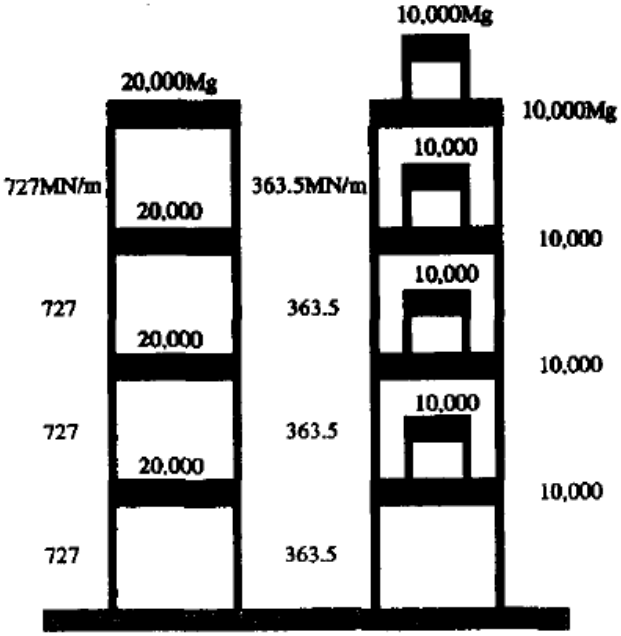
(a) *(b)* *(c)* ***(d)***

- Passive response control systems: *(b)* seismic isolation, *(c)* energy dissipation devices, ***(d)*** Passive TMDs **(linear, easy to design)**

BUT: Require the attachment of very large attached masses

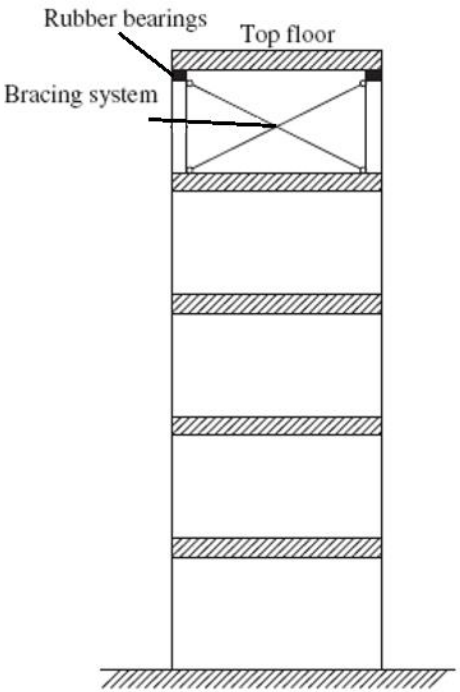
Motivation for the tuned mass-damper-inerter (TMDI)

Multiple TMDs

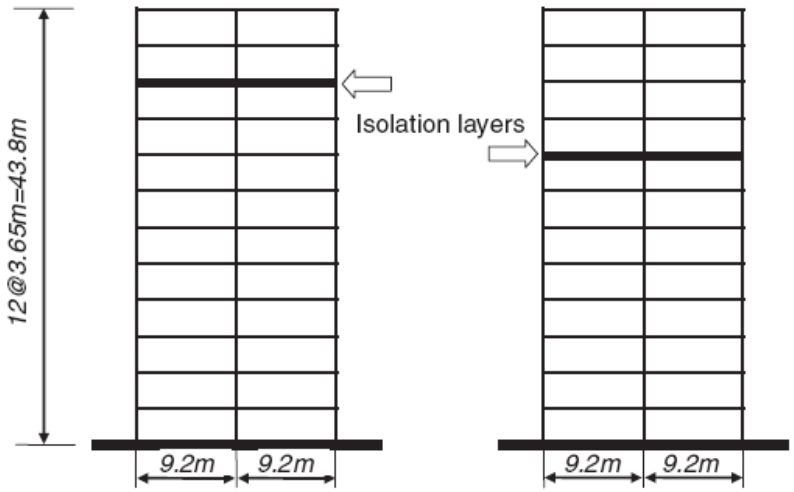


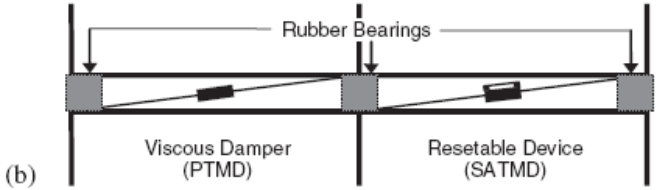
Feng/Mita, *JEM,* (1996)

Large-mass and Non-linear TMDs



Moutinho, *EESD* (2012)





De Angelis/Perno/Reggio, *EESD* (2012)

Need for a TMD-based passive dynamic vibration absorber for earthquake engineering applications that:

- does not involve excessively large attached mass

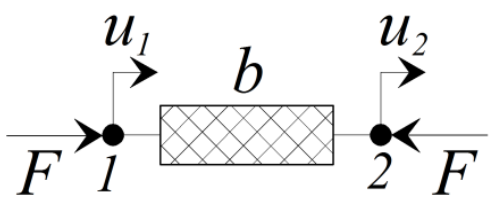
-is linear

-is amenable to the standard TMD optimum design approaches

Motivation for the tuned mass-damper-inerter (TMDI)

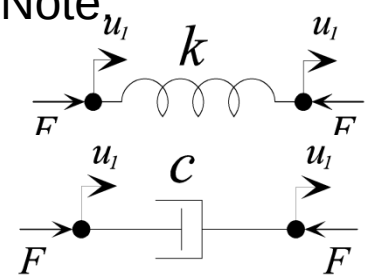
➢ The concept of the inerter and its mass amplification effect

- The inerter is a linear device with two terminals free to move independently and develops an internal (resisting) force proportional to the relative acceleration of its terminals (Smith 2002)



$$F = b(\ddot{u}_1 - \ddot{u}_2)$$

Note:

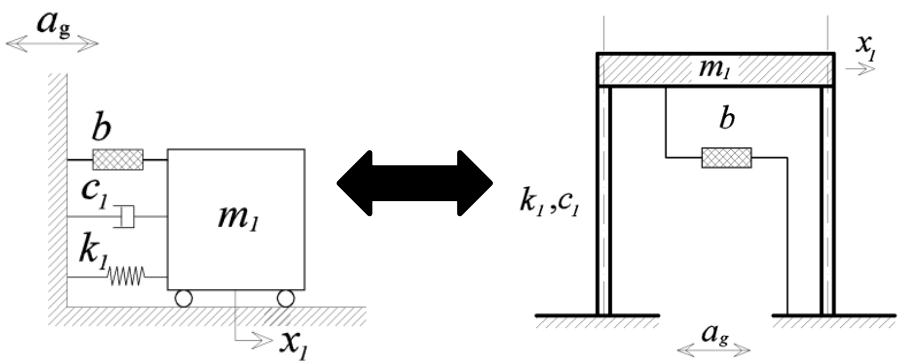


$$F = k(u_1 - u_2)$$

$$F = c(\dot{u}_1 - \dot{u}_2)$$

- The constant of proportionality ***b*** ("inertance") is in mass units (kg)

- Single-degree of freedom oscillator connected to the ground via an inerter:



$$(m_1 + b)\ddot{x}_1 + c_1\dot{x}_1 + k_1 x_1 = -m_1 a_g$$

- The inerter increases the m_1 mass:

$$m_1 \longrightarrow m_1 + b$$

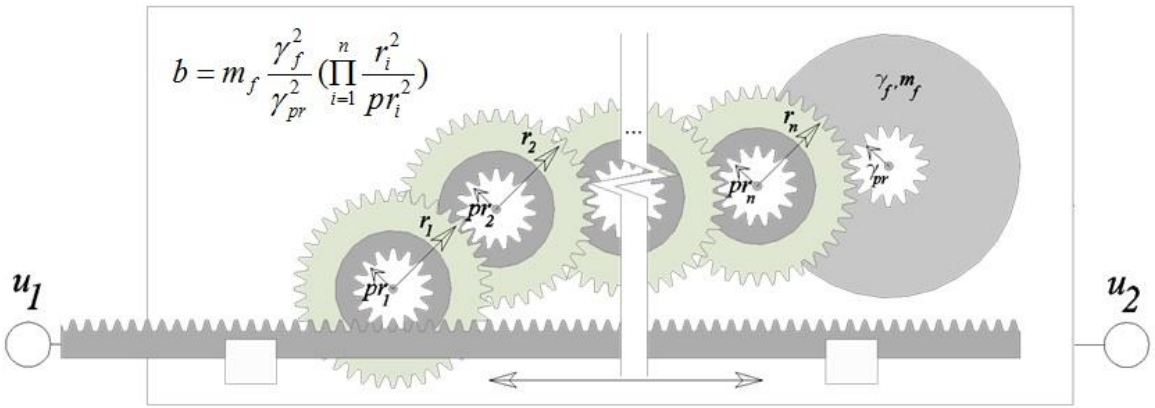
"mass amplification effect"

Smith, *IEEE* (2002); Takewaki/Murakami/Yoshitomi/Tsuji, *SCHM* (2012); Chen/Hu/Huang/Chen, *JSV* (2014)

Motivation for the tuned mass-damper-inerter (TMDI)

- **Flywheel-based inerter with rack and pinion mechanism** Smith, *IEEE* (2002)

- Assume a mechanical realisation of the inerter comprising a plunger that drives a rotating flywheel through a rack, pinion and gearing system:



$$F = b(\ddot{u}_1 - \ddot{u}_2), \quad b = m_f \frac{\gamma_f^2}{p\gamma_f^2} (\prod_{i=1}^{n} \frac{r_i^2}{pr_i^2})$$

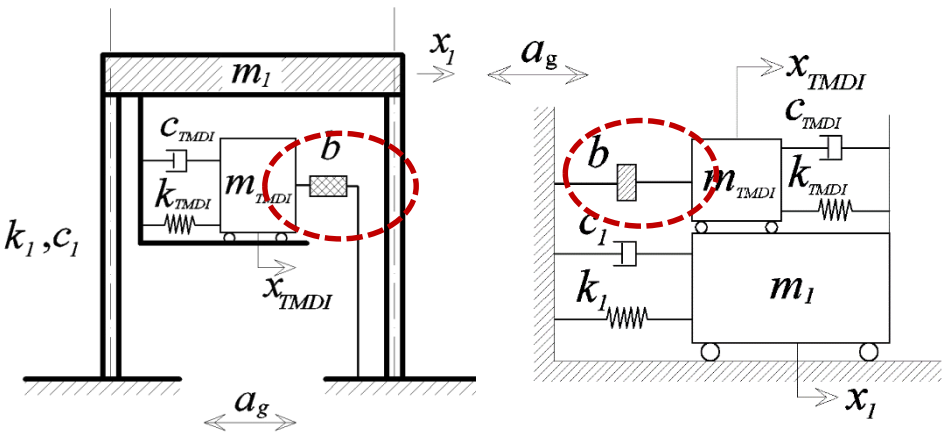
- The constant of proportionality *b* (inertance) - mass units (>> physical mass of device)
- Inerter can have an inertance ("apparent" mass) orders of magnitude higher than its physical mass.

The tuned mass-damper-inerter (TMDI)

A linear potentially lighter than the TMDI passive control solution with the deisgn simplicity of the TMD Marian/Giaralis, *PBEE,* (2013, 2014)

SDOF primary structures

- Mass attached to the primary structure via a linear spring and a viscous damper + an inerter device which connects the attached mass to the ground.



MDOF primary structures

- TMDI as an inter-story connective device placed in a 'diagonal' configuration.

For *b=0:*

Tuned mass-damper (TMD)

For m_{TMDI}=0:

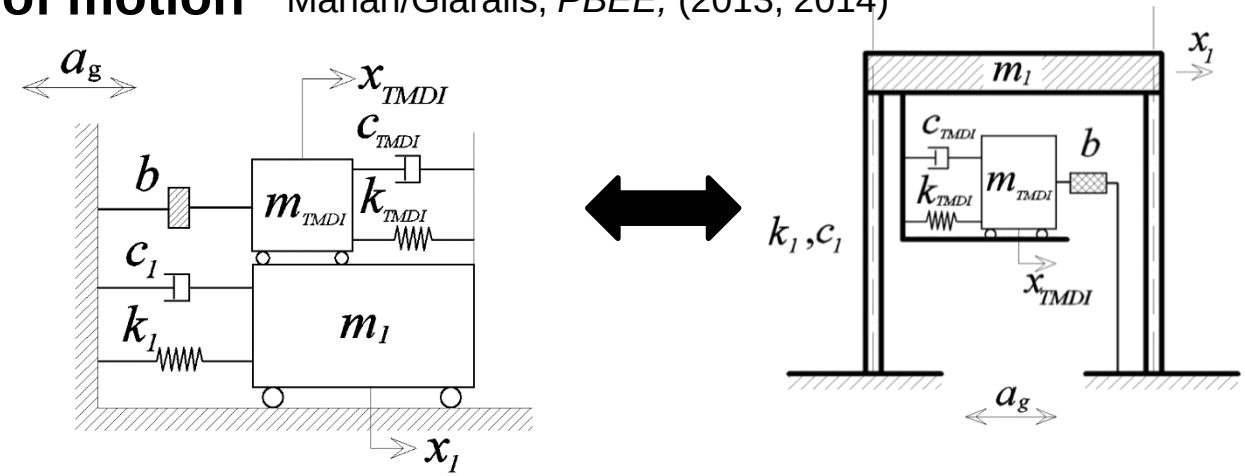
Tuned inerter-damper (TID)

Lazar/Neild/Wagg, *EESD* (2013, 2014)

TMDI for SDOF primary structures

➢ Equations of motion

Marian/Giaralis, *PBEE*, (2013, 2014)



Time domain:

$$\begin{bmatrix} m_{TMDI}+b & 0 \\ 0 & m_1 \end{bmatrix}\begin{bmatrix} \ddot{x}_{TMDI} \\ \ddot{x}_1 \end{bmatrix}+\begin{bmatrix} c_{TMDI} & -c_{TMDI} \\ -c_{TMDI} & c_1+c_{TMD} \end{bmatrix}\begin{bmatrix} \dot{x}_{TMDI} \\ \dot{x}_1 \end{bmatrix}+\begin{bmatrix} k_{TMDI} & -k_{TMDI} \\ -k_{TMDI} & k_1+k_{TMDI} \end{bmatrix}\begin{bmatrix} x_{TMDI} \\ x_1 \end{bmatrix}=-\begin{bmatrix} m_{TMDI} \\ m_1 \end{bmatrix}a_g$$

$$\omega_{TMDI}=\sqrt{\frac{k_{TMDI}}{m_{TMDI}+b}} \qquad \zeta_{TMDI}=\frac{c_{TMDI}}{2(m_{TMDI}+b)\omega_{TMDI}} \qquad \mu=\frac{m_{TMDI}}{m_1} \qquad \upsilon_{TMDI}=\frac{\omega_{TMDI}}{\omega_1} \qquad \beta=\frac{b}{m_1}$$

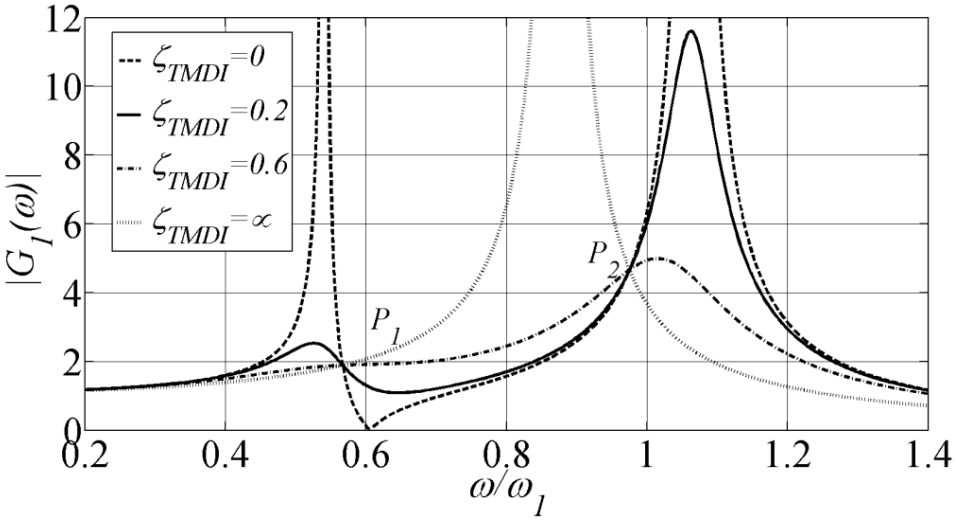
Frequency domain:

$$G_1(\omega)=\frac{x_1}{a_g}\omega_1^{\,2}=\frac{(1+\mu)\omega_{TMDI}^{\;2}-\omega^2+i2\zeta_{TMDI}(1+\mu)\omega_{TMDI}\omega}{\left(1-\frac{\omega^2}{\omega_1^{\,2}}\right)\left(\omega_{TMDI}^{\;2}-\omega^2+i2\zeta_{TMDI}\omega_{TMDI}\omega\right)-\frac{\beta+\mu}{\omega_1^{\,2}}\left(\omega_{TMDI}^{\;2}+i2\zeta_{TMDI}\omega_{TMDI}\omega\right)\omega^2}$$

➢ Optimum design for harmonic excitation

Marian/Giaralis, *SCHM,* (2014, 2016)

- **Design problem:** given an undamped primary system with specific dynamic characteristics (m_1,k_1,$c_1=0$), determine the TMDI parameters (k_{TMDI}, c_{TMDI}) which minimise the response of the primary system, given m_{TMDI} and b.



-mass ratio μ=0.1,
-inertance ratio $\beta=0.1$,
-frequency ratio $\upsilon_{TMDI}=0.5$.

- **Solution -** 'Equal points' design method: there exist two fixed points where the FRF curves intersect (noted P_1 and P_2), independent of c_{TMDI} (ζ_{TMDI}).
- Optimum response is obtained if and only if there exists two local maxima and both have the same amplitude

➢ Optimum design for harmonic excitation Den Hartog approach

- Minimum response <=> (if and only if) $|G_1(\omega)|$ has two local maxima with equal amplitudes at the stationary points P_1 and P_2.
- Enforce:

1) $|G_1(\omega)|$ is independent of ζ_{TMDI}:

$$\lim_{\zeta_{TMDI}\to\infty}|G_1(\omega)|^2 = \lim_{\zeta_{TMDI}\to 0}|G_1(\omega)|^2$$

2) Equal amplitude at points P_1 and P_2 for the limit $\zeta_{TMDI}\to\infty$:

$$\lim_{\zeta_{TMDI}\to\infty}|G_1(\omega_{P1})| = \lim_{\zeta_{TMDI}\to\infty}|G_1(\omega_{P2})|$$

3) $|G_1(\omega)|$ is maximized locally at points P_1 and P_2:

$$\left.\frac{\partial|G_1(\omega)|}{\partial\upsilon}\right|_{\omega=\omega_{P1}} = \left.\frac{\partial|G_1(\omega)|}{\partial\upsilon}\right|_{\omega=\omega_{P2}} = 0$$

- Optimum frequency ratio υ_{TMDI}:

$$\upsilon_{TMDI} = \frac{1}{1+\beta+\mu}\sqrt{\frac{(1+\mu)(2-\mu)-\mu\beta}{2(1+\mu)}}$$

- Optimum TMDI damping ratio ζ_{TMDI}:

$$\zeta_{TMDI} = \sqrt{\frac{\beta^2\mu+6\mu(1+\mu)^2+\beta(1+\mu)(6+7\mu)}{8(1+\mu)(1+\beta+\mu)[2+\mu(1-\beta-\mu)]}}$$

- Dynamic amplification factor:

$$\max_{\omega}\{|G_1(\omega)|\} = \sqrt{\frac{(1+\mu)(\beta+2\mu+2)}{\beta+\mu}}$$

=> For b=0 (β=0)
optimum TMD design is retrieved

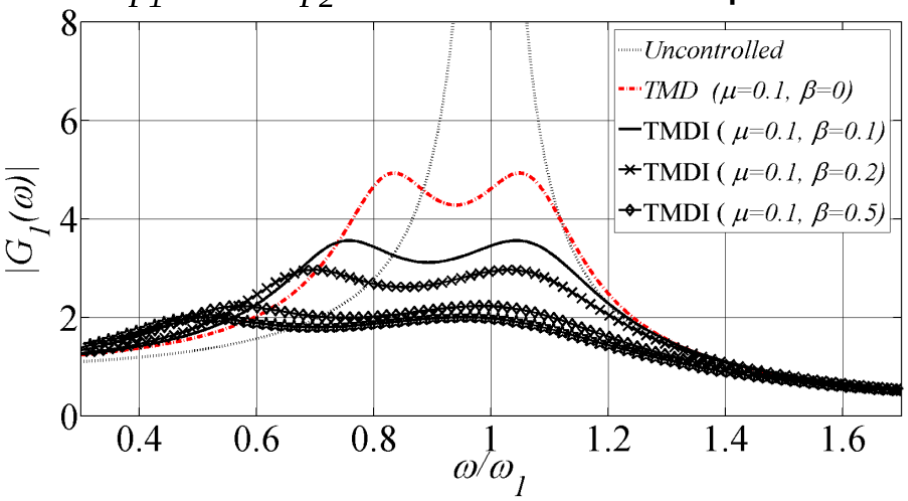
Marian/Giaralis, *SCHM,* (2014, 2016)

TMDI for SDOF primary structures

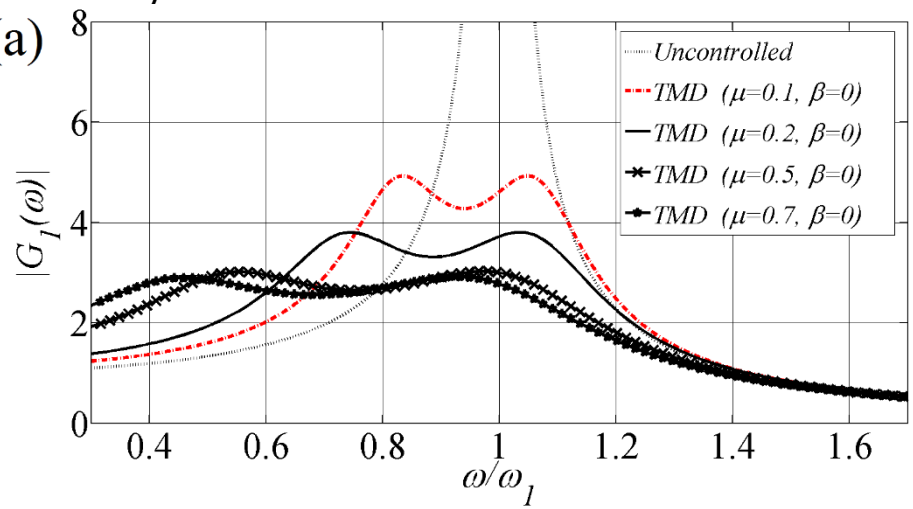
➢ Optimum design for harmonic excitation: vibration suppression

Marian/Giaralis, *SCHM,* (2014, 2016)

- All FRF curves attain two local maxima of equal height at the frequencies ω_{P1} and ω_{P2} whose location depend on the ratio β.



(a)



For the same m_{TMDI} mass, the inclusion of the inerter (TMDI vs. TMD):

- reduces the peak response of the primary structure (at ω_1, ω_{P1} and ω_{P2}),

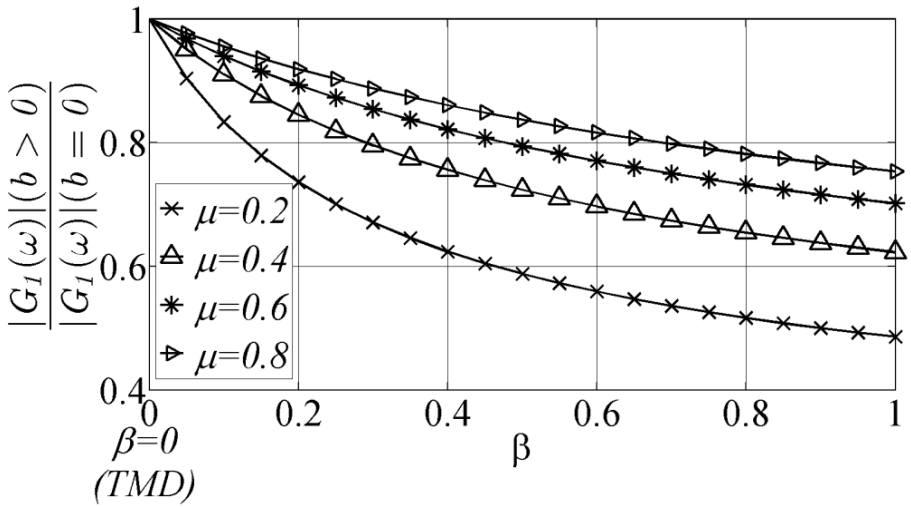
- increases robustness to "detuning effects" - Large β values - wider range of frequencies peak response reduction compared to the classical TMD.

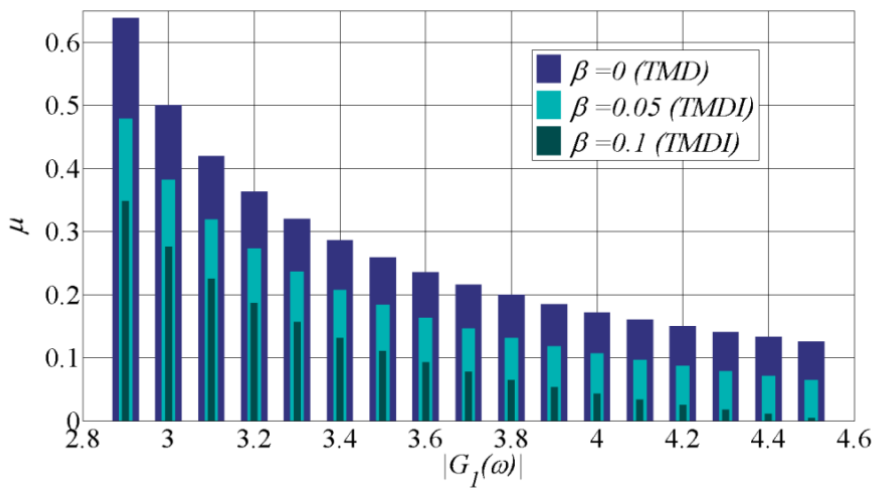
- Practically, the inerter furnishes all the positive effects of increasing the attached mass without the negative effect of the added weight.

TMDI for SDOF primary structures

➢ Optimum design for harmonic excitation: weight reduction

- Peak response amplitude for optimally designed TMDI normalized by the peak response amplitude of the optimally designed classical TMD ($\beta = b = 0$).





- TMDI reduction saturates as the ratio β increases

Marian/Giaralis, *SCHM,* (2014, 2016)

- Incorporation of the inerter to the classical TMD system is more effective for vibration suppression for smaller attached masses m_{TMDI}.

Example: **- $|G_1(\omega)|$ = 2.9 if:**

1) TMD solution: mass ratio $\mu = 0.63$

2)TMDI solution - mass ratio $\mu = 0.34$ (for $\beta = 0.1$)

➢ Optimum design for white noise excitation

- Closed form expressions for TMDI parameters **to minimize the variance of the relative displacement** of undamped primary structures

$$\sigma_1^{\,2} = \int_{-\infty}^{+\infty} \left|G_1(\omega)\right|^2 S(\omega) d\omega \qquad S(\omega)=S_0 \text{ (ideal white noise)}$$

Impose: $\dfrac{\partial \sigma_1^2}{\partial \upsilon} = 0$ and $\dfrac{\partial \sigma_1^2}{\partial \zeta_{TMD}} = 0 \Rightarrow$

TMDI

$$\upsilon_{TMDI} = \frac{1}{1+\beta+\mu} \frac{\sqrt{[\beta(\mu-1)+(2-\mu)(1+\mu)]}}{\sqrt{2(1+\mu)}}$$

$$\zeta_{TMDI} = \frac{\sqrt{(\beta+\mu)}\sqrt{\beta(3-\mu)+(4-\mu)(1+\mu)}}{2\sqrt{2(1+\beta+\mu)[\beta(1-\mu)+(2-\mu)(1+\mu)]}}$$

$$\sigma_{1,\min}^{\;2} = \pi S_0 \omega_1 (1+\mu) \sqrt{\frac{(1+\mu)[\beta(3-\mu)+(4-\mu)(1+\mu)]}{(\beta+\mu)(1+\beta+\mu)}}$$

=> For b=0 (β=0)

TMD

$$\upsilon_{TMD} = \frac{\sqrt{(1-\mu/2)}}{1+\mu}$$

$$\zeta_{TMD} = \frac{\sqrt{\mu(1-\mu/4)}}{\sqrt{4(1+\mu)(1-\mu/2)}}$$

$$\sigma_{1,\min}^{\;2} = \pi S_0 \omega_1 (1+\mu) \sqrt{\frac{(1+\mu)(4-\mu)}{\mu}}$$

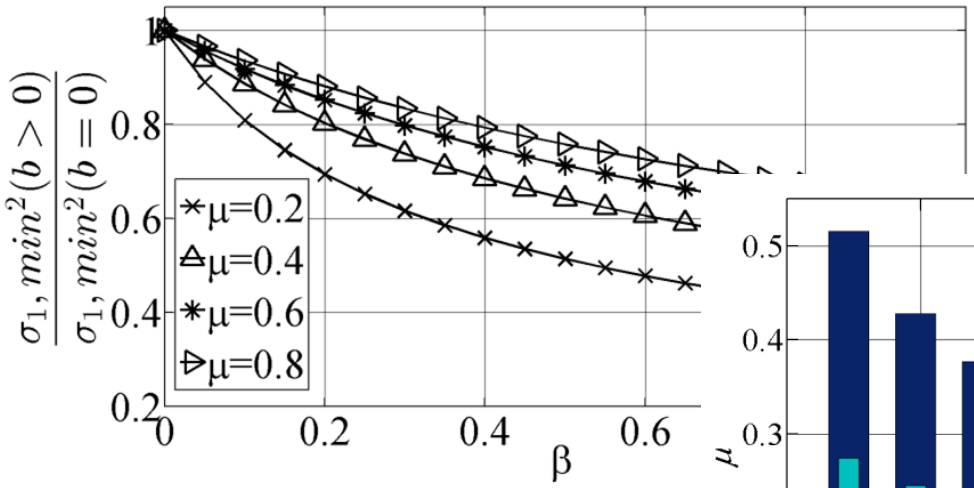
Marian/Giaralis, *PEM,* (2014)

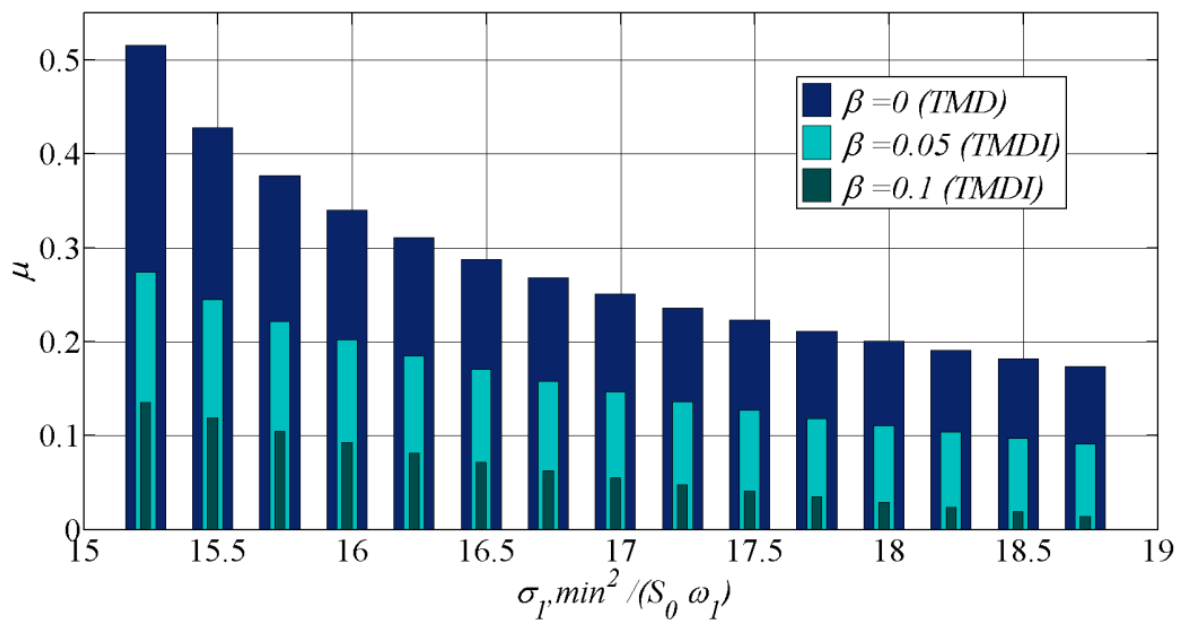
Warburton, *EESD,* (1982)

TMDI for SDOF primary structures

➢ Optimum design for white noise excitation

- Additional m_{TMDI} mass values (coefficient μ)required for achieving the same level of performance in terms of displacement response variance for white noise base excitationfor the proposed TMDI configuration ($\beta>0$) and classical TMD (β=0)





Marian/Giaralis, *PEM,* (2014)

TMDI for MDOF primary structures

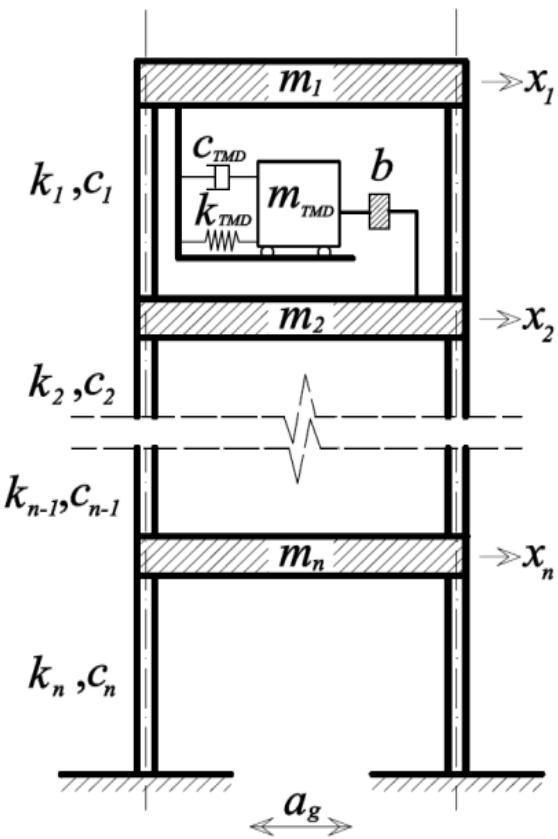
- **Equations of motion:** N-storey TMDI equipped shear frame building structure
  Marian/Giaralis, *PEM* (2013,2014)

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{M}\boldsymbol{\delta} a_g$$

$$\mathbf{x} = \{x_{TMD}(t) \quad x_1(t) \quad x_2(t) \quad \cdots \quad x_n(t)\}^T$$

- Displacements vector

$$\boldsymbol{\delta} = \{1 \quad 1 \quad 1 \quad \cdots \quad 1\}^T$$



$$\mathbf{M} = \begin{bmatrix} m_{TMDI}+b & 0 & -b & 0 & \cdots & 0 \\ 0 & m_1 & 0 & 0 & \cdots & \vdots \\ -b & 0 & m_2+b & 0 & \cdots & \vdots \\ 0 & 0 & 0 & m_3 & \cdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & \cdots & \cdots & \cdots & 0 & m_n \end{bmatrix}$$

- Mass matrix

$$\mathbf{C} = \begin{bmatrix} c_{TMDI} & -c_{TMDI} & 0 & \cdots & 0 \\ -c_{TMDI} & c_1 + c_{TMDI} & -c_1 & 0 & \vdots \\ 0 & -c_1 & c_1 + c_2 & -c_2 & \vdots \\ 0 & 0 & -c_2 & \ddots & 0 \\ \vdots & \vdots & \vdots & \vdots & -c_{n-1} \\ 0 & \cdots & 0 & -c_{n-1} & c_{n-1} + c_n \end{bmatrix}$$

- Damping matrix

$$\mathbf{K} = \begin{bmatrix} k_{TMDI} & -k_{TMDI} & 0 & \cdots & 0 \\ -k_{TMDI} & k_1 + k_{TMDI} & -k_1 & 0 & \vdots \\ 0 & -k_1 & k_1 + k_2 & -k_2 & \vdots \\ 0 & 0 & -k_2 & \ddots & 0 \\ \vdots & \vdots & \vdots & \vdots & -k_{n-1} \\ 0 & \cdots & 0 & -k_{n-1} & k_{n-1} + k_n \end{bmatrix}$$

- Stiffness matrix

TMDI for MDOF primary structures

➢ Optimum design for evolutionary stochastic (seismic) excitation

Marian/Giaralis, *PEM* (2014)

Given:

- **A primary structure** with specific dynamic characteristics,
- An inerter with **pre-specified inertance *b***,
- **A pre-specified TMD mass**:

$$\mu = \frac{m_{TMDI}}{M_1} \qquad M_1 = \{\phi_{1n}\}^T \left[M_p\right] \{\phi_{1n}\}$$

Optimize for:

•TMD parameters : damping ratio (ζ_{TMDI}) and frequency ratio:

$$\zeta_{TMDI} = \frac{c_{TMDI}}{2(m_{TMDI} + b)\omega_{TMDI}} \qquad \upsilon_{TMDI} = \frac{\omega_{TMDI}}{\omega_1}$$

Considering the objective function

$$PI = J^{TMDI} / J^0, \qquad J^{TMDI} = \int_0^\infty \left|G_1(\omega)\right|^2 \max_t \{S(\omega,t)\} d\omega$$

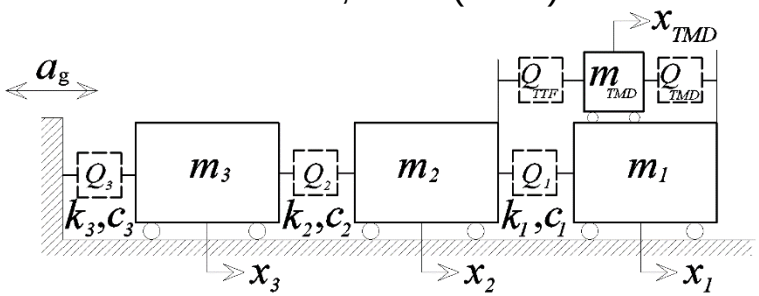
S(ω,t) – evolutionary power spectral density (EPSD) function modelling the support excitation modelled by a stationary stochastic process

$|G_1(\omega)|^2$ - squared modulus of the frequency response function corresponding to peak floor displacement

TMDI for MDOF primary structures

➢ Optimum design for evolutionary stochastic (seismic) excitation

Marian/Giaralis, *PEM* (2014)



- Equivalent mechanical model of the proposed frame structure. Mechanical admittance formulation.
- Mechanical admittances

$$Q_1(s) = \frac{k_1}{s} + c_1,\ Q_2(s) = \frac{k_2}{s} + c_2,\ Q_3(s) = \frac{k_3}{s} + c_3,\ Q_{TMD}(s) = \frac{k_{TMD}}{s} + c_{TMD}, \quad \text{and} \quad Q_{TTF}(s) = bs$$

- Equations of motion in Laplace form:

$$[B]\{\tilde{X}\} = -[M]\{\delta\}\tilde{\mathrm{A}},$$

- Admittance matrix *[B]*:

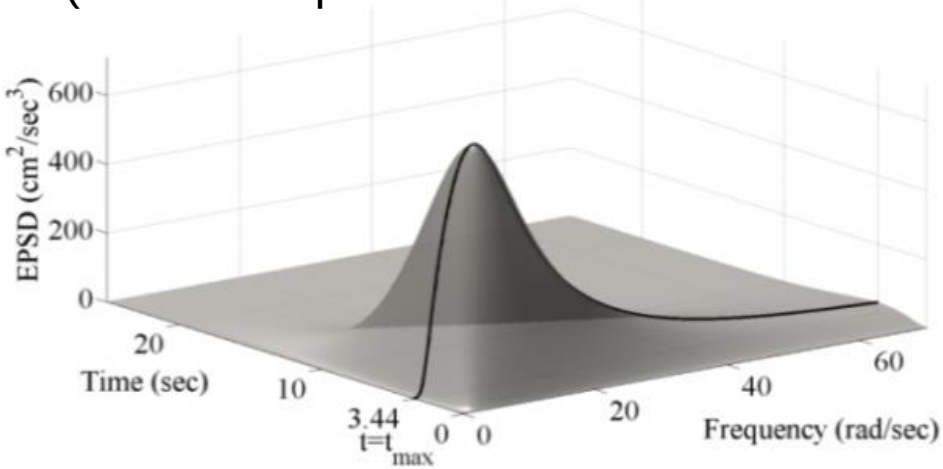
$$[B] = \begin{bmatrix} m_1 s^2 + [Q_1(s) + Q_{TMD}(s)]s & -Q_1(s)s & 0 & -Q_{TMD}(s) \\ -Q_1(s)s & m_2 s^2 + [Q_1(s) + Q_2(s) + Q_{TTF}(s)]s & -Q_2(s)s & -Q_{TTF}(s) \\ 0 & -Q_2(s)s & m_3 s^2 + [Q_2(s) + Q_3(s)]s & 0 \\ -Q_{TMD}(s) & -Q_{TTF}(s) & 0 & m_{TMD} s^2 + [Q_{TMD}(s) + Q_{TTF}(s)]s \end{bmatrix}$$

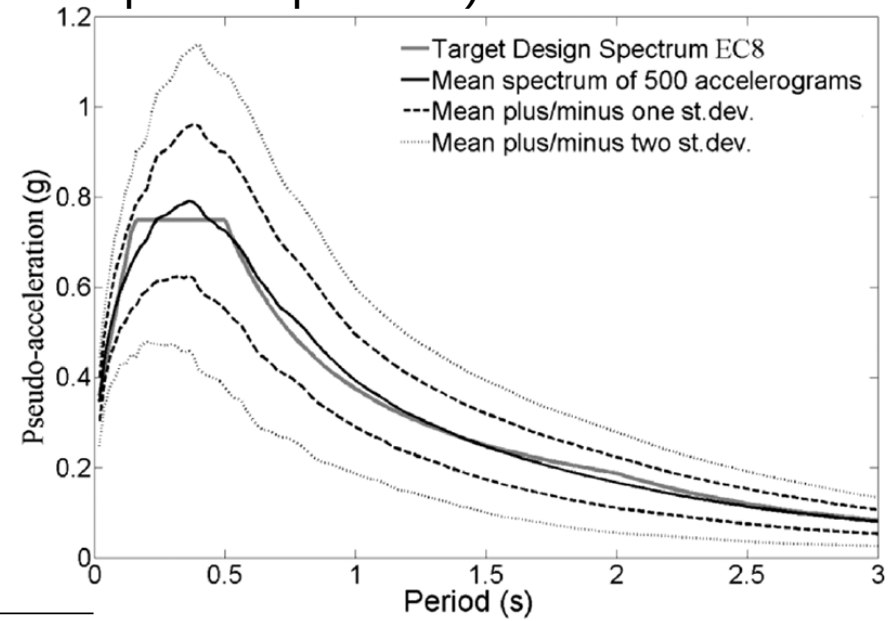
- Compute overall system transfer function: $G_1(s) = \dfrac{\tilde{x}_1(s)}{\tilde{\mathrm{A}}(s)}$,

➢ Considered input evolutionary stochastic seismic excitation

Modeling of the seismic excitation
(EPSD compatible with the Eurocode 8 elastic response spectrum)

Giaralis/Spanos, *E&S* (2012)





$$S(\omega,t) = C\,t\exp\left(\frac{-bt}{2}\right)\frac{1+4\zeta_g^2\left(\frac{\omega}{\omega_g}\right)^2}{\left(1-\left(\frac{\omega}{\omega_g}\right)^2\right)^2+4\zeta_g^2\left(\frac{\omega}{\omega_g}\right)^2}\,\frac{\left(\frac{\omega}{\omega_f}\right)^4}{\left(1-\left(\frac{\omega}{\omega_f}\right)^2\right)^2+4\zeta_g^2\left(\frac{\omega}{\omega_f}\right)^2}$$

C (cm/sec$^{2.5}$)	b (1/sec)	ζ_g	ω_g (rad/sec)	ζ_f	ω_f (rad/sec)
17.76	0.58	0.78	10.73	0.90	2.33

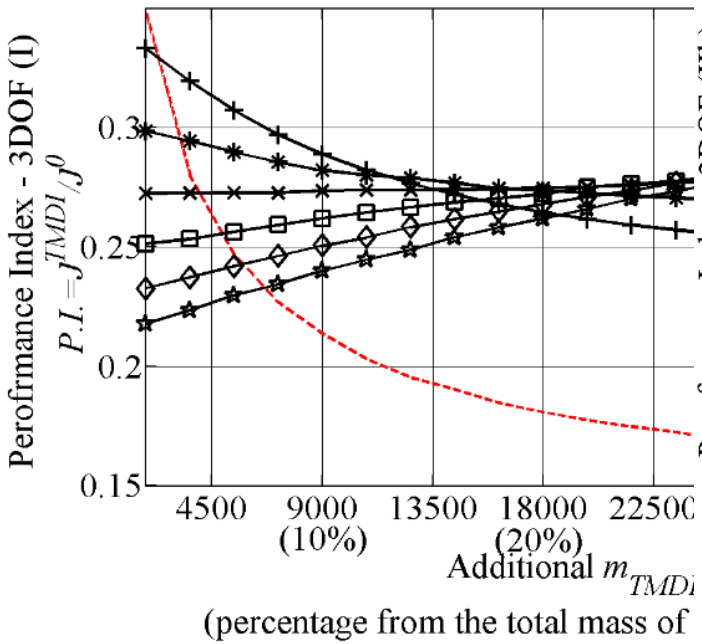
Range of different 3-storey building frames

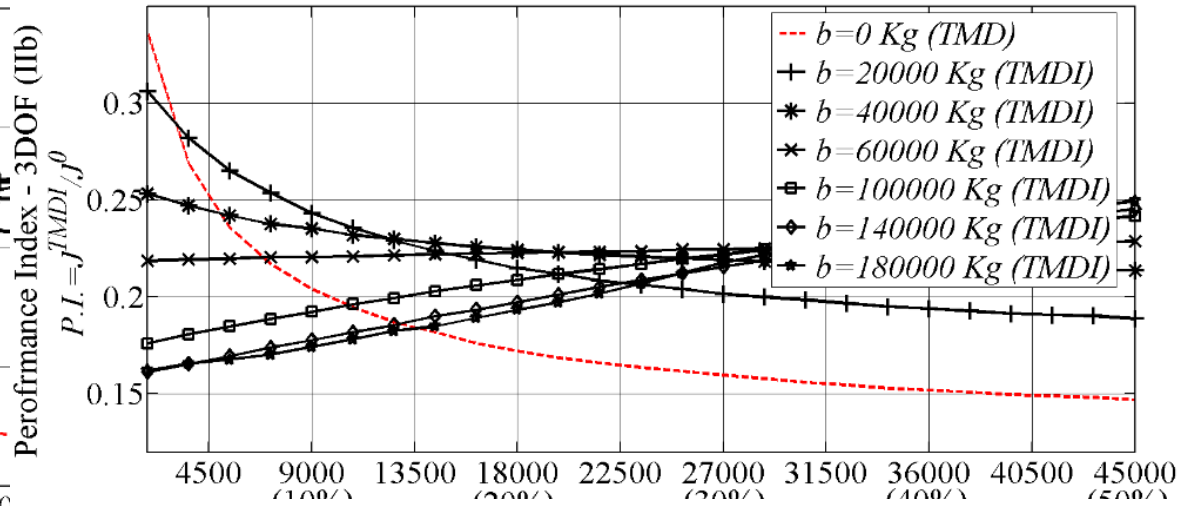
Structure	Story	Mass (kg)	Stiffness (N/m)
REGULAR IN ELEVATION			
3DOF **(I)**	1 (top)	30 x 10^3	30 x 10^6
	2	30 x 10^3	30 x 10^6
	3	30 x 10^3	30 x 10^6
WITH REDUCED TOP FLOOR STIFFNES			
3DOF **(IIa)**	1 (top)	30 x 10^3	25 x 10^6
	2	30 x 10^3	30 x 10^6
	3	30 x 10^3	30 x 10^6
3DOF **(IIb)**	1 (top)	30 x 10^3	20 x 10^6
	2	30 x 10^3	30 x 10^6
	3	30 x 10^3	30 x 10^6
WITH INCREASED TOP FLOOR MASS			
3DOF **(IIIa)**	1 (top)	35 x 10^3	30 x 10^6
	2	30 x 10^3	30 x 10^6
	3	30 x 10^3	30 x 10^6
3DOF **(IIIb)**	1 (top)	40 x 10^3	30 x 10^6
	2	30 x 10^3	30 x 10^6
	3	30 x 10^3	30 x 10^6

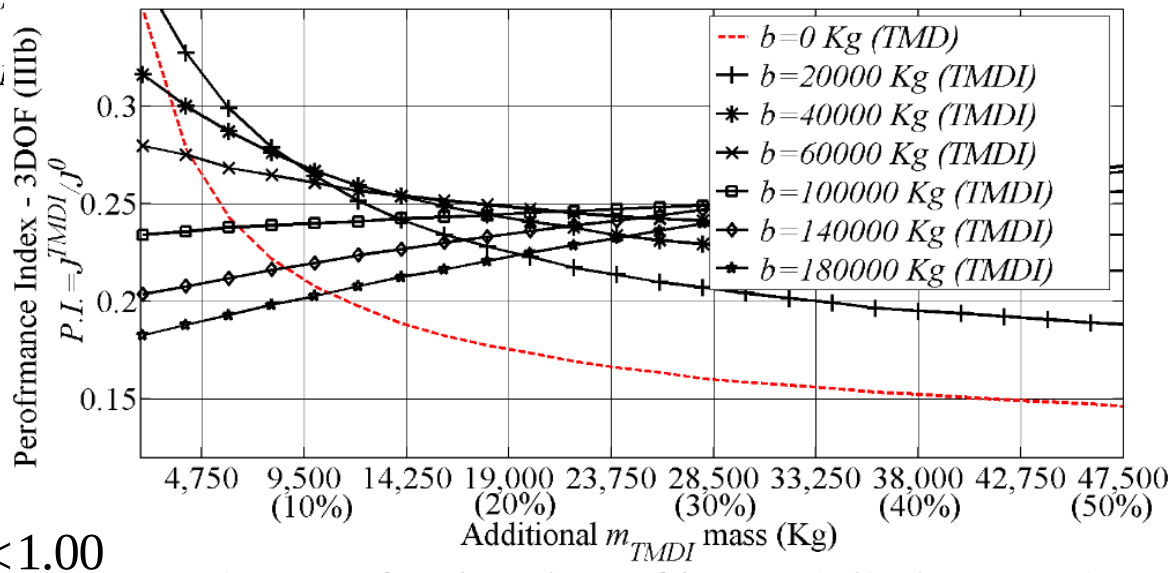
Structure	Mode	Period (s)	Frequency (rad/s)
REGULAR IN ELEVATION			
3DOF **(I)**	1st	0.44	14.07
	2nd	0.16	39.43
	3rd	0.11	56.98
WITH REDUCED TOP FLOOR STIFFNES			
3DOF **(IIa)**	1st	0.45	13.92
	2nd	0.17	37.40
	3rd	0.11	55.45
3DOF **(IIb)**	1st	0.46	13.69
	2nd	0.18	34.82
	3rd	0.02	54.16
WITH INCREASED TOP FLOOR MASS			
3DOF **(IIIa)**	1st	0.47	13.47
	2nd	0.16	38.42
	3rd	0.11	56.56
3DOF **(IIIb)**	1st	0.49	12.93
	2nd	0.17	37.62
	3rd	0.11	56.27

Eurocode 8 compatible seismic design of TMDI equipped building structures

➢ Performance in terms of top floor displacement variance





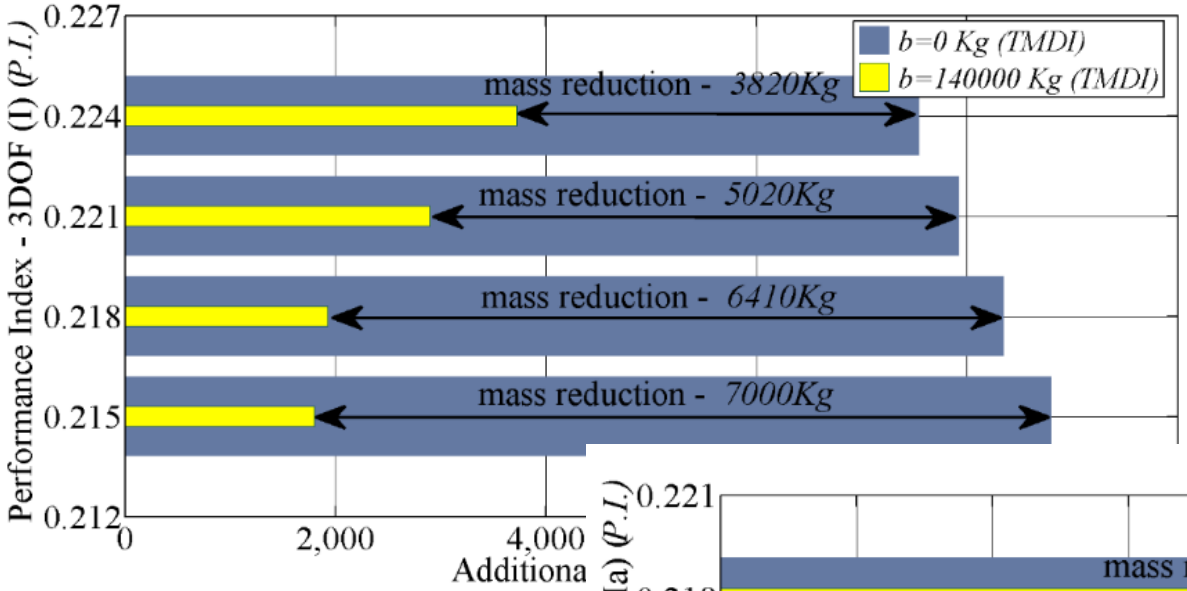


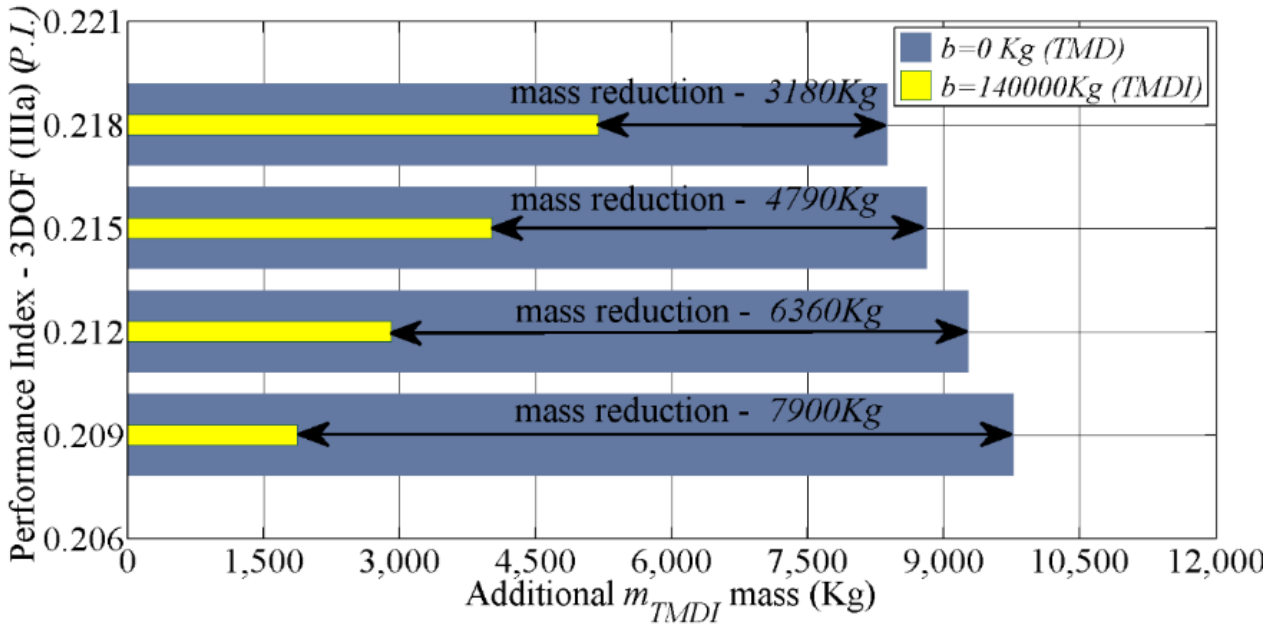
<u>Optimisation method</u>:

- MATLAB® built-in “min-max” constraint optimization algorithm employing a sequential programming method

$$0.5 < \upsilon_{TMDI} < 1.10 \;\; and \;\; 0 < \zeta_{TMDI} < 1.00$$

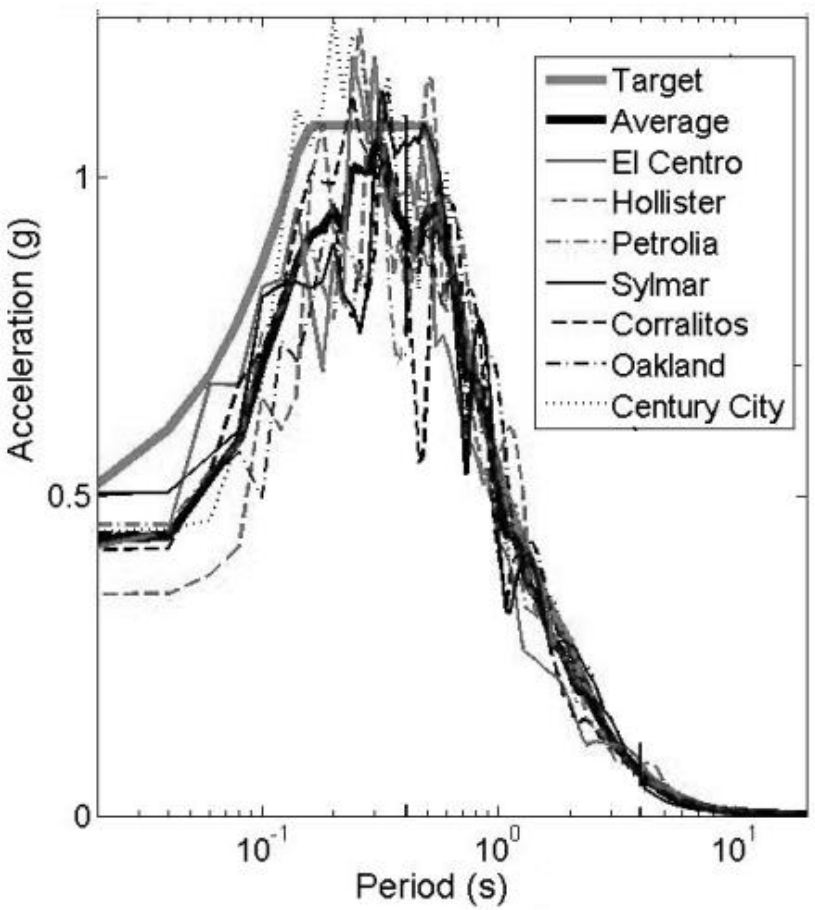
➢ Weight reduction





Eurocode 8 compatible seismic assessment of TMDI equipped building structures

➢ TMDI assessment using Eurocode 8 compatible accelerograms



Giaralis/Spanos, *SDEE* (2009)

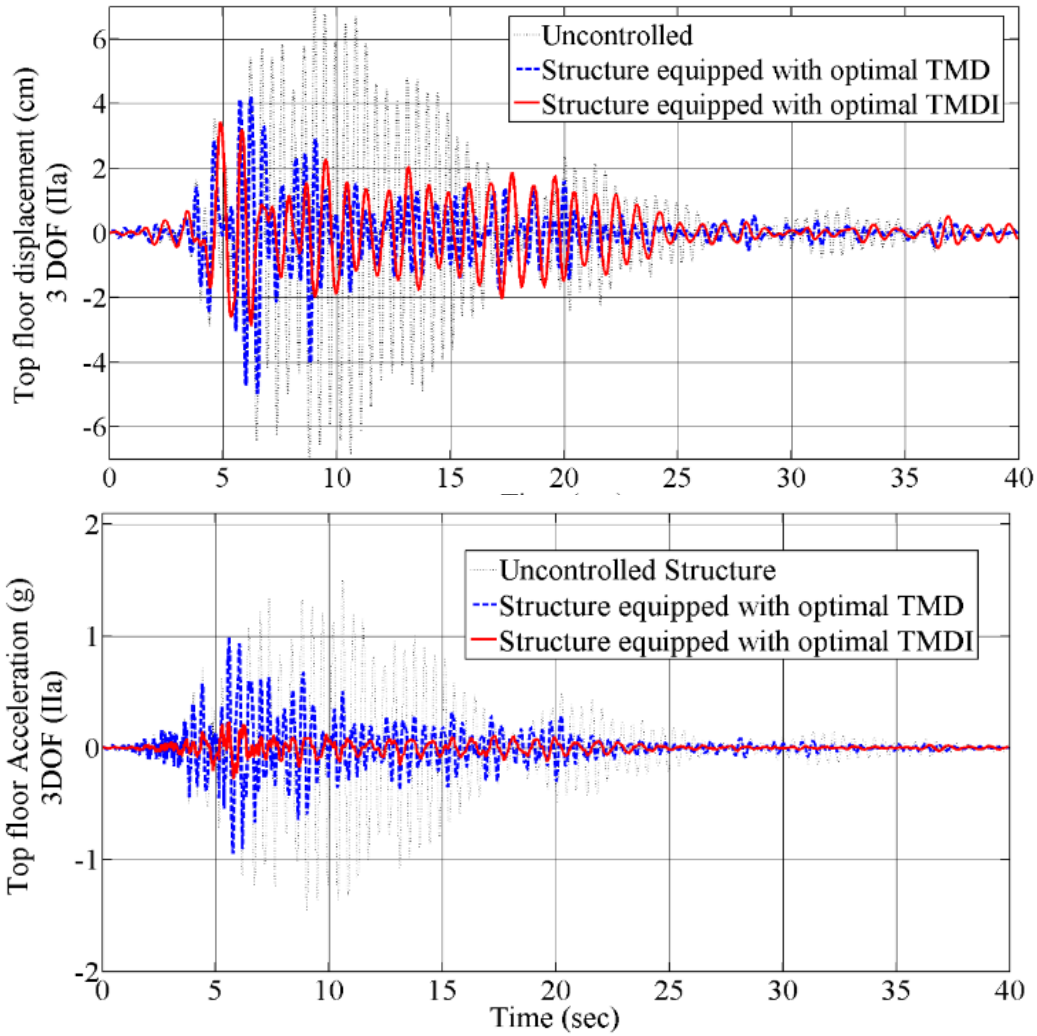


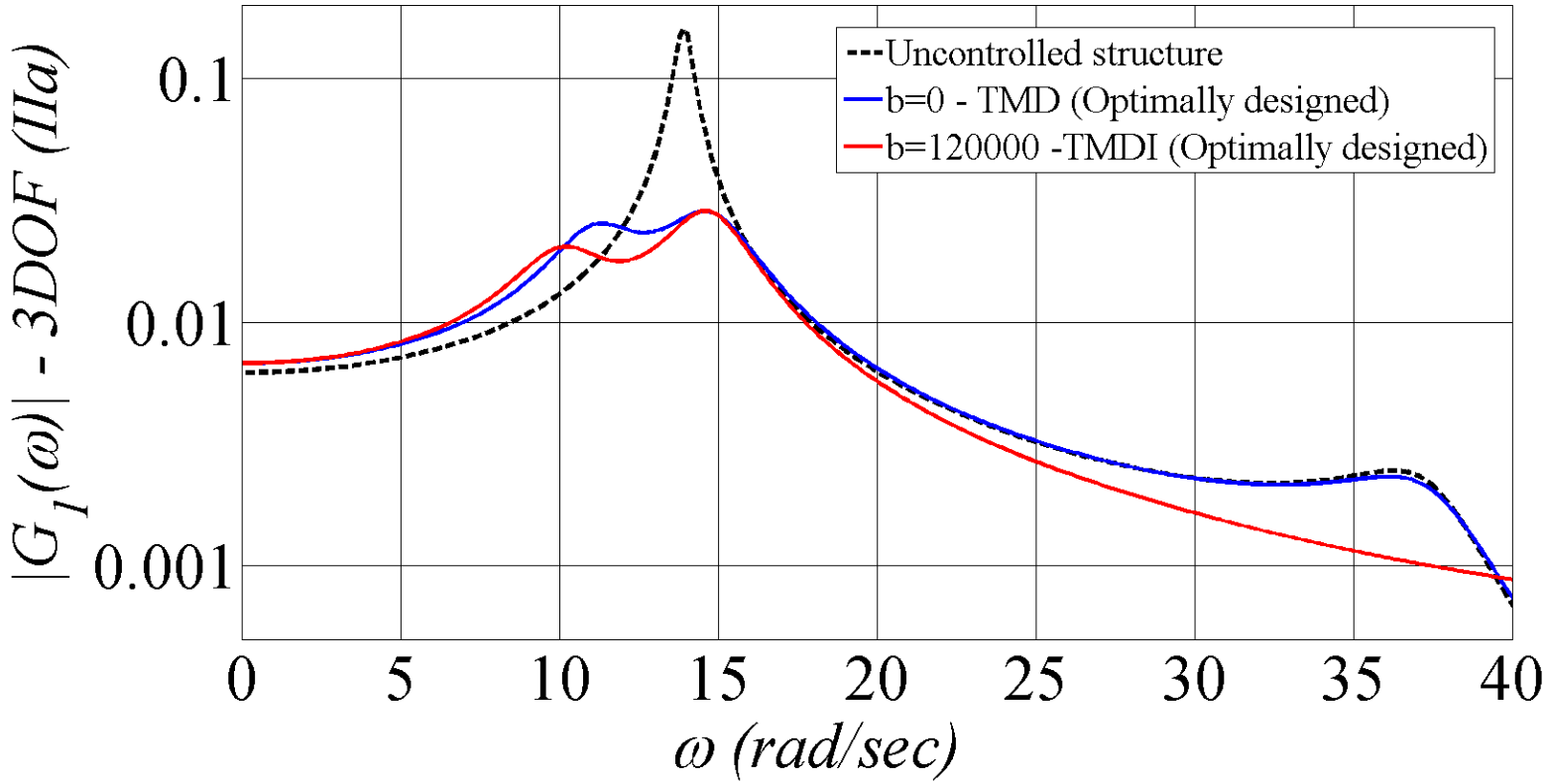
Table 5. Maximum top floor displacements (cm)

	Ground acceleration no.	#1	#2	#3	#4	#5	#6	#7	Average
3 DOF Structure (IIa)	**Primary structure**	8.21	10.68	8.28	9.87	9.91	7.60	7.85	***8.91***
	TMD (m=5400 Kg, b=0 Kg)	4.86	4.79	5.51	6.38	5.01	4.97	5.65	**5.30**
	TMDI (m=5400 Kg, b=80000 Kg)	5.54	5.09	4.72	5.23	4.43	5.52	6.68	**5.31**
	TMDI (m=5400 Kg, b=100000 Kg)	4.08	3.91	4.15	4.38	7.18	3.78	3.58	**4.44**

Table 6. Maximum top floor acceleration (*g*)

	Ground acceleration no.	#1	#2	#3	#4	#5	#6	#7	Average
3 DOF Structure (IIa)	**Primary structure**	1.59	2.25	1.53	1.97	1.97	1.50	1.58	***1.77***
	TMD (m=5400 Kg, b=0 Kg)	1.07	0.97	0.92	1.22	1.42	0.99	1.12	**1.10**
	TMDI (m=540 Kg, b=80000 Kg)	0.57	0.44	0.44	0.40	0.34	0.45	0.55	**0.46**
	TMDI (m=5400 Kg, b=100000 Kg)	0.33	0.35	0.38	0.31	0.37	0.32	0.30	**0.34**
	TMDI (m=5400 Kg, b=120000 Kg)	0.32	0.31	0.33	0.30	0.24	0.27	0.31	**0.30**

The TMDI suppresses the higher modes of vibration, not just the fundamental mode as the TMD does…



TMDI for MDOF primary structures

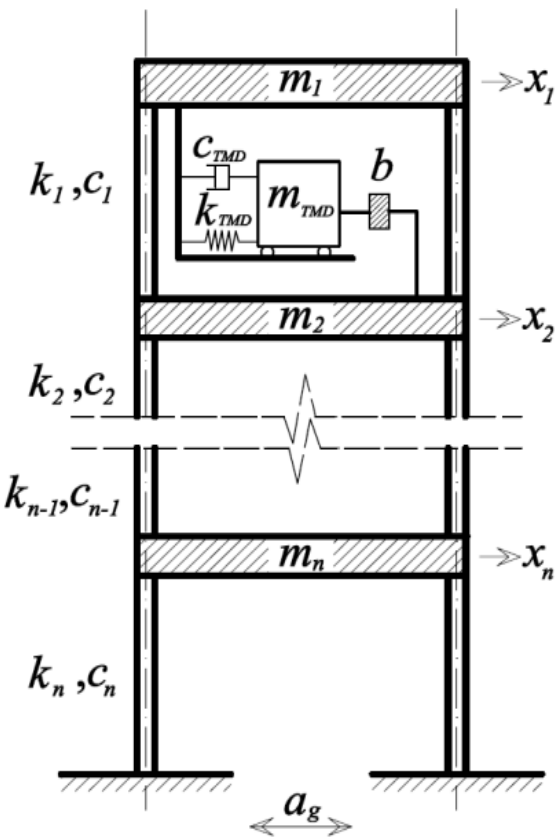
- **Equations of motion:** N-storey TMDI equipped shear frame building structure
  Marian/Giaralis, *PEM* (2013,2014)

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{M}\boldsymbol{\delta} a_g$$

$$\mathbf{x} = \{x_{TMD}(t) \quad x_1(t) \quad x_2(t) \quad \cdots \quad x_n(t)\}^T$$

- Displacements vector

$$\boldsymbol{\delta} = \{1 \quad 1 \quad 1 \quad \cdots \quad 1\}^T$$



$$\mathbf{M} = \begin{bmatrix} m_{TMDI}+b & 0 & -b & 0 & \cdots & 0 \\ 0 & m_1 & 0 & 0 & \cdots & \vdots \\ -b & 0 & m_2+b & 0 & \cdots & \vdots \\ 0 & 0 & 0 & m_3 & \cdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & \cdots & \cdots & \cdots & 0 & m_n \end{bmatrix}$$

- Mass matrix

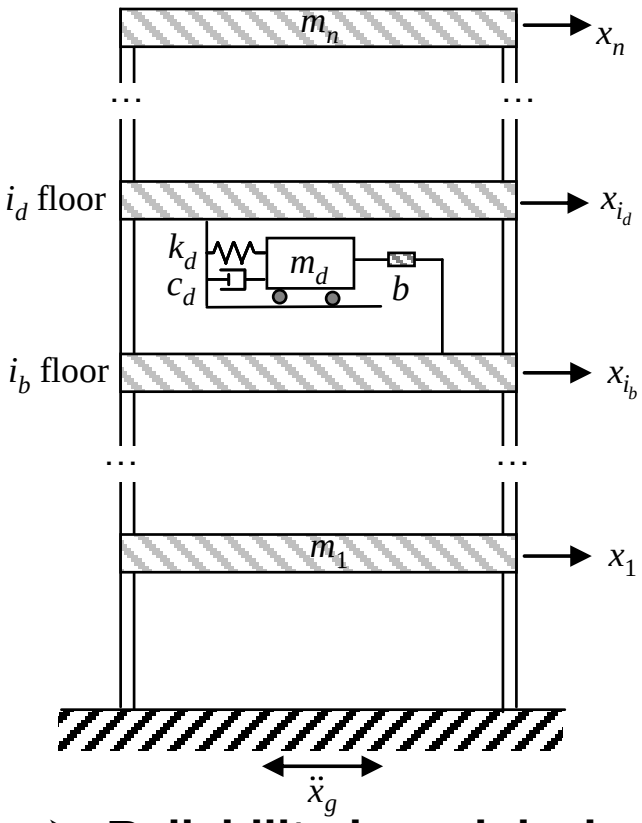
$$\mathbf{C} = \begin{bmatrix} c_{TMDI} & -c_{TMDI} & 0 & \cdots & 0 \\ -c_{TMDI} & c_1 + c_{TMDI} & -c_1 & 0 & \vdots \\ 0 & -c_1 & c_1 + c_2 & -c_2 & \vdots \\ 0 & 0 & -c_2 & \ddots & 0 \\ \vdots & \vdots & \vdots & \vdots & -c_{n-1} \\ 0 & \cdots & 0 & -c_{n-1} & c_{n-1} + c_n \end{bmatrix}$$

- Damping matrix

$$\mathbf{K} = \begin{bmatrix} k_{TMDI} & -k_{TMDI} & 0 & \cdots & 0 \\ -k_{TMDI} & k_1 + k_{TMDI} & -k_1 & 0 & \vdots \\ 0 & -k_1 & k_1 + k_2 & -k_2 & \vdots \\ 0 & 0 & -k_2 & \ddots & 0 \\ \vdots & \vdots & \vdots & \vdots & -k_{n-1} \\ 0 & \cdots & 0 & -k_{n-1} & k_{n-1} + k_n \end{bmatrix}$$

- Stiffness matrix

Reliability-based optimum design framework for TMDI equipped seismically excited building structures



- **Alternative connectivity arrangements**

$$\left(\mathbf{M}_s + \mathbf{R}_d m_d \mathbf{R}_d^T + \mathbf{R}_c b \mathbf{R}_c^T\right)\ddot{\mathbf{x}}_s + \left(m_d \mathbf{R}_d + b\mathbf{R}_c\right)\ddot{y}$$
$$+ \mathbf{C}_s \dot{\mathbf{x}}_s + \mathbf{K}_s \mathbf{x}_s = -\left(\mathbf{M}_s + \mathbf{R}_d m_d \mathbf{R}_d^T\right)\mathbf{R}_s \ddot{x}_g$$

and

$$(m_d + b)\ddot{y} + \left(m_d \mathbf{R}_d^T + b\mathbf{R}_c^T\right)\ddot{\mathbf{x}}_s + c_d \dot{y} + k_d y = -m_d \mathbf{R}_d^T \mathbf{R}_s \ddot{x}_g$$

$\mathbf{R}_d$: TMD location vector

$\mathbf{R}_b$: inerter location vector

$\mathbf{R}_c = \mathbf{R}_d - \mathbf{R}_b$: connectivity vector

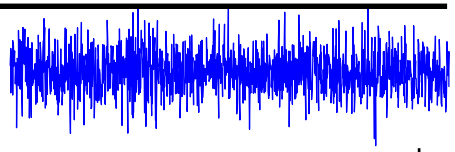
- **Reliability-based design accounting for higher modes contribution:** Minimize the probability that any interstorey drift, or total floor acceleration, or the attached mass displacement (stroke) outcrosses a predefined threshold within the duration of stationary seismic excitation

Giaralis/Taflanidis, (2015)

- **Inertance b is treated as a free/design parameter**
- **Seismic input and structural uncertainty can be accounted for**

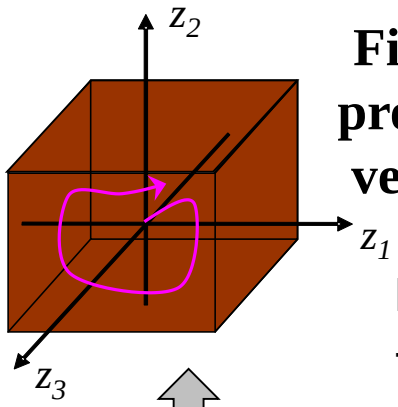
Reliability-based optimum design framework for TMDI equipped seismically excited building structures

$$P_F(\boldsymbol{\varphi}) = P\left(\mathbf{z}(t) \notin D_s \text{ for some } t \in [0,T]\right)$$



white noise (input)

$$\dot{\mathbf{x}}(t) = \mathbf{A}(\boldsymbol{\varphi})\mathbf{x}(t) + \mathbf{E}(\boldsymbol{\varphi})\mathbf{w}(t)$$



First-passage probability for vector output

Performance variables
-interstorey drifts
-total floor accelerations
-attached mass stroke

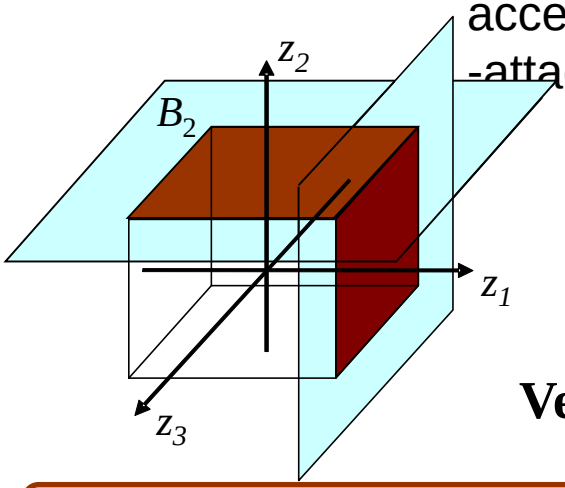
$$\mathbf{z}(t) = \mathbf{C}(\boldsymbol{\varphi})\mathbf{x}(t)$$

Design variables

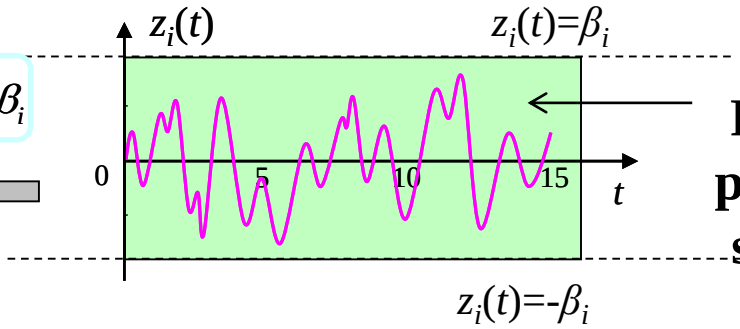
$$\boldsymbol{\varphi} = \{b\ \omega_d\ \zeta_d\}$$

Augmented state-space matrix

output



$$B_i : |z_i(t)| = \beta_i$$



Vector-output

First-passage probability for scalar output

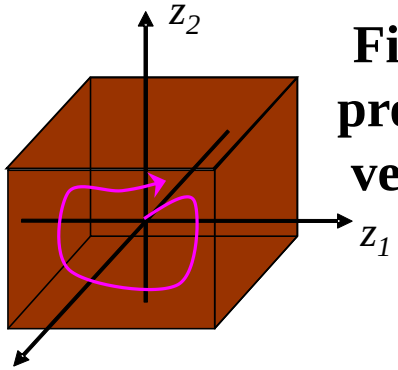
$$D_s : \left\{\mathbf{z}(t) \in \mathbb{R}^{n_z} : |z_i(t)| < \beta_i, \forall i = 1,\ldots,n_z\right\}$$

"Hyper-rectangular in the performance variables space

Giaralis/Taflanidis, (2015)

Reliability-based optimum design framework for TMDI equipped seismically excited building structures

$$P_F(\boldsymbol{\varphi}) = P\left(\mathbf{z}(t) \notin D_s \text{ for some } t \in [0,T]\right) = 1 - \exp\left(-\nu_{\mathbf{z}}^{+}(\boldsymbol{\varphi})T\right)$$



First-passage probability for vector output

Assumption: Stationary excitation and response

Reliability-based design

$$\boldsymbol{\varphi}^* = \arg\min_{\boldsymbol{\varphi}\in\Phi} P_F(\boldsymbol{\varphi}) = \arg\min_{\boldsymbol{\varphi}\in\Phi} \nu_{\mathbf{z}}^{+}(\boldsymbol{\varphi})$$

Unconditional outcrossing rate for vector output

$$\nu_{\mathbf{z}}^{+}(\boldsymbol{\varphi}) = \lim_{\Delta t\to 0} \frac{E[\text{number of out-crossings in } [t, t+\Delta t] \mid \text{no out-crossings in } [0,t]]}{\Delta t}$$

$$\approx \sum_{i=1}^{n_z} P_{z_i}(\boldsymbol{\varphi})\lambda_{z_i}(\boldsymbol{\varphi})r_{z_i}^{+}(\boldsymbol{\varphi})$$

Rice's ***conditional*** outcrossing rate for ***scalar*** output

Correction factor for ***vector*** output, considering correlation of outcrossing's at different parts of boudnary

Temporal correlation factor correction for ***unconditional*** outcrossing rate

Giaralis/Taflanidis, (2015)

➢ Properties of the 10-storey TMDI equipped frame structure

Stories	Storey Mass (ton)	Storey Stiffness $\tilde{k}_i$ (MN/m)	Mode	Period (s)	Participation factor
1-4	900	782.22	1st	1.50	81.7%
5-7	900	626.10	2nd	0.55	11.8%
8-10	900	469.57	3rd	0.33	3.7%

Nominal modal damping: 3.5%

Seismic Excitation: Clough-Penzien spectrum

$$S_g(\omega) = s_o \frac{\omega_g^4 + 4\zeta_g^2\omega^2\omega_g^2}{\left(\omega_g^2 - \omega^2\right)^2 + 4\zeta_g^2\omega_g^2\omega^2} \frac{\omega^4}{\left(\omega_f^2 - \omega^2\right)^2 + 4\zeta_f^2\omega_f^2\omega^2}$$

with mean values ω_g=3π, ω_f=π/2, ζ_g=0.4, ζ_f=0.8, a_{RMS}=0.09g

Giaralis/Taflanidis, (2015)

Adopted thresholds

Interstorey drifts: 3.3cm
Floor acceleration: 0.5g
Stroke: 1m

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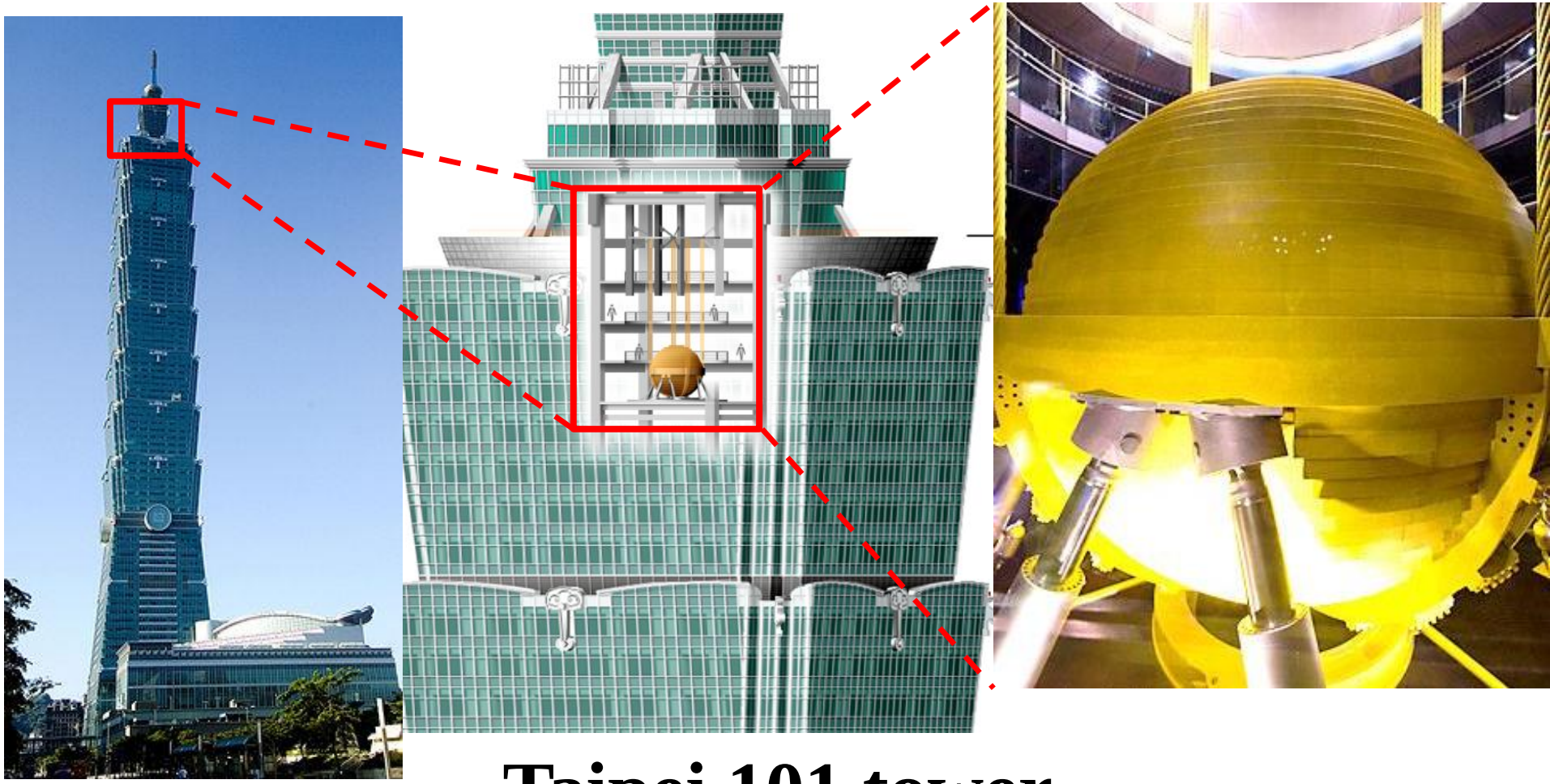
Optimal out-crossing rate $\nu_z^+(\varphi^*)\cdot 100$ and optimal inerter ratio β % (in parenthesis) for different TMDI topologies. "TMD" denotes the cases for which β=0 is optimal

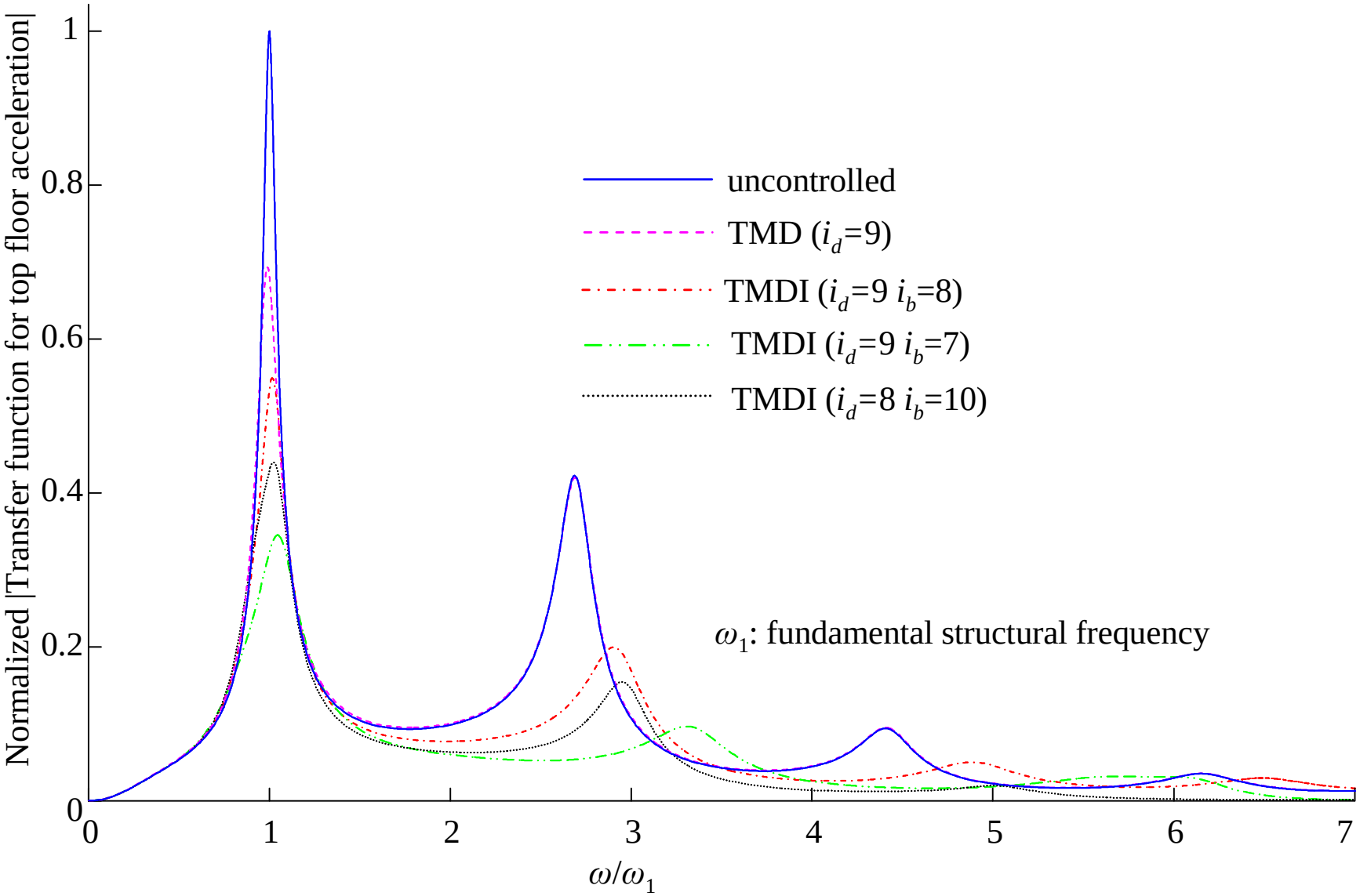
Topology		mass ratio μ_d									
i_d	i_b	0.1%	0.3%	0.5%	0.75%	1%	1.5%	2%	3%	4%	5%
10	9	1.507 (217.7)	1.497 (210.9)	1.487 (204.7)	1.161 (TMD)	0.795 (TMD)	0.381 (TMD)	0.194 (TMD)	0.064 (TMD)	0.028 (TMD)	0.014 (TMD)
10	8	0.348 (122.2)	0.342 (120.1)	0.335 (119.7)	0.327 (114.2)	0.319 (110.1)	0.302 (104.6)				
9	10	1.526 (224.7)	1.554 (230.8)	1.581 (238.34)	1.249 (TMD)	0.881 (TMD)	0.448 (TMD)	0.244 (TMD)	0.093 (TMD)	0.047 (TMD)	0.027 (TMD)
9	8	0.626 (234.7)	0.625 (228.9)	0.623 (223.3)	0.620 (216.4)	0.616 (209.8)					
9	7	0.071 (139.6)	0.071 (136.8)	0.071 (134.7)	0.070 (133.2)	0.070 (129.5)	0.069 (125.9)	0.068 (120.8)	0.066 (111.5)		
8	9	0.634 (240.63)	0.648 (251.0)	0.662 (260.3)	0.677 (341.9)	0.691 (345.8)	0.565 (TMD)	0.331 (TMD)	0.145 (TMD)	0.083 (TMD)	0.057 (TMD)
8	10	0.355 (126.5)	0.365 (133.2)	0.373 (151.2)	0.382 (152.4)	0.392 (153.3)	0.411 (156.3)	0.331 (TMD)			
8	7	0.276 (315.85)	0.279 (309.7)	0.281 (303.5)	0.284 (296.4)	0.287 (296.5)	0.292 (277.7)	0.297 (266.5)			
8	6	0.024 (180.27)	0.025 (176.6)	0.025 (173.2)	0.025 (169.9)	0.026 (167.1)	0.027 (161.9)	0.027 (157.4)	0.028 (148.5)	0.029 (139.6)	0.030 (129.6)
7	8	0.278 (326.6)	0.285 (341.9)	0.291 (413.9)	0.299 (416.7)	0.306 (420.1)	0.322 (424.9)	0.338 (429.7)	0.204 (TMD)	0.117 (TMD)	0.079 (TMD)
7	9	0.072 (143.7)	0.074 (151.3)	0.076 (171.8)	0.079 (172.5)	0.082 (174.6)	0.087 (176.8)	0.093 (179.4)	0.105 (184.1)	0.118 (188.7)	
7	6	0.135 (418.5)	0.137 (413.9)	0.139 (410.4)	0.141 (404.3)	0.144 (403.6)	0.148 (394.0)	0.153 (387.2)	0.163 (376.6)	0.117 (TMD)	

Feasibility of inerters spanning multiple floors

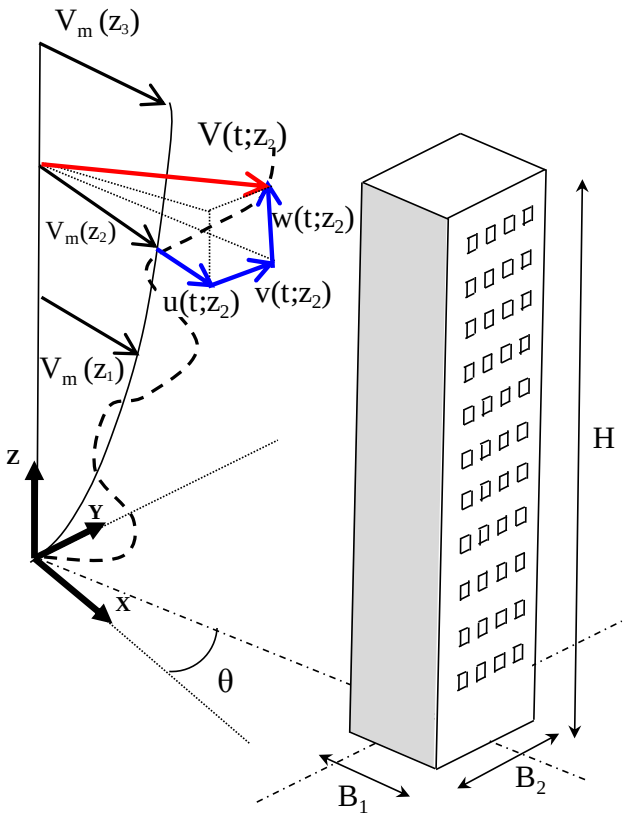


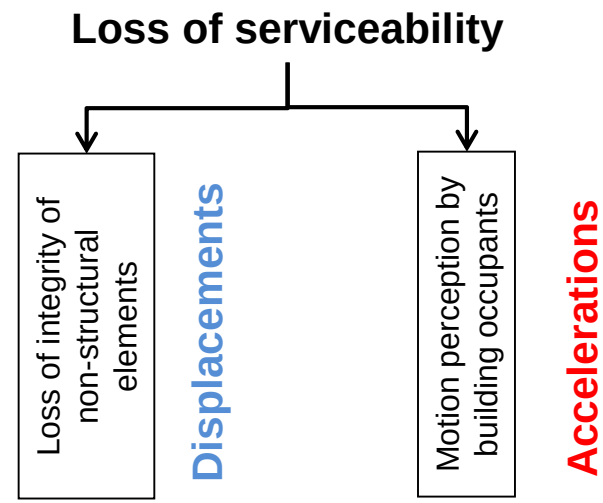
Taipei 101 tower

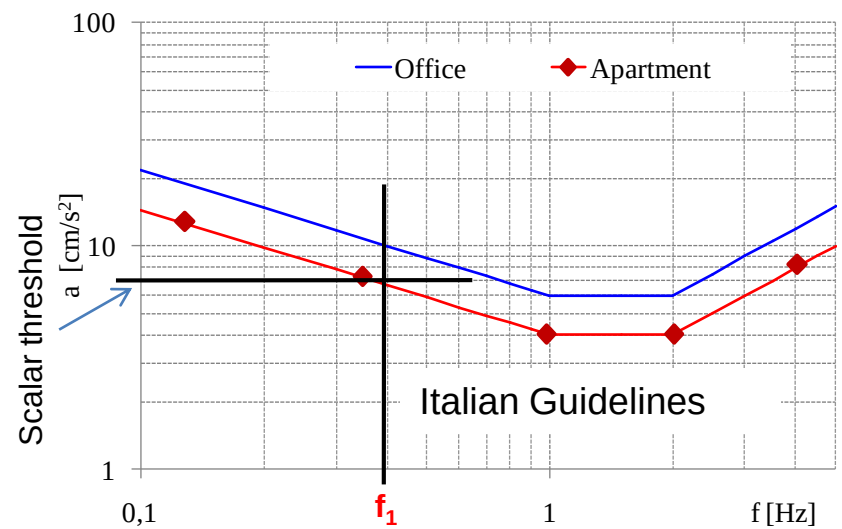
Effectiveness of the TMDI to suppress higher modes



Serviceability in wind-excited high-rise (tall) buildings







1 Discomfort level in terms of perception thresholds

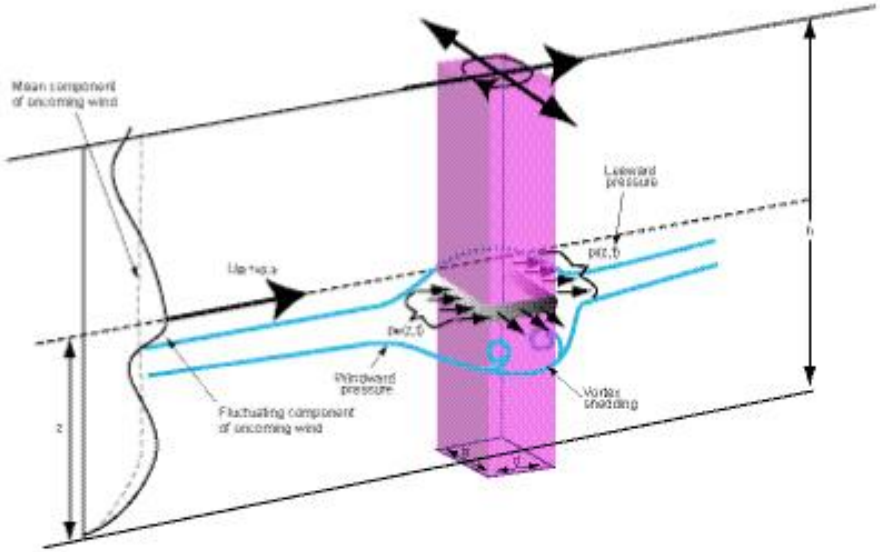
2 Usually cross-wind vibration is critical for **occupants comfort**

3 The reference period for comfort evaluation is 1 year

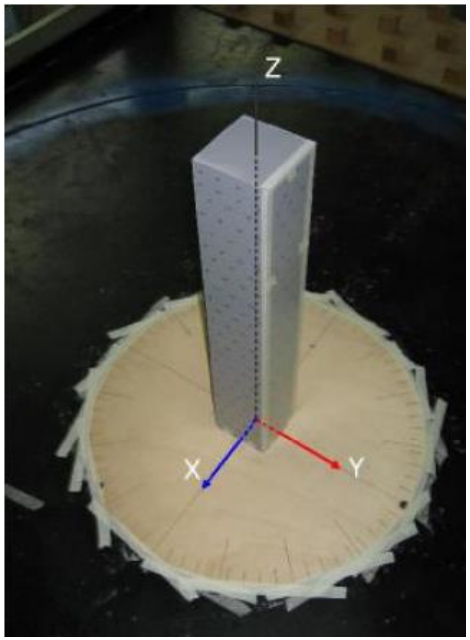
4 **1st natural frequency is dominant**

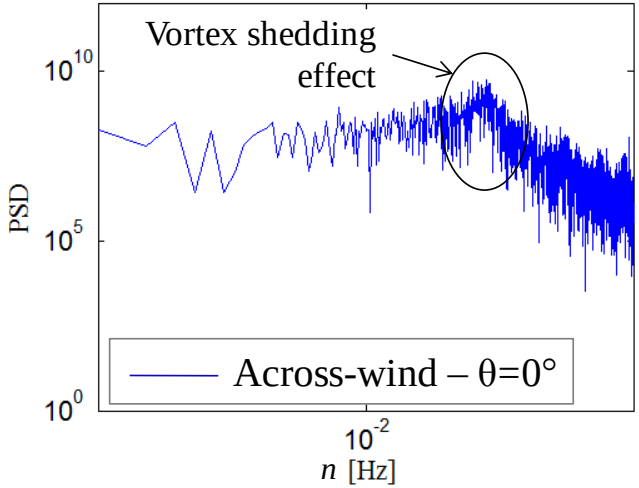
Cross-wind forces due to vortex shedding

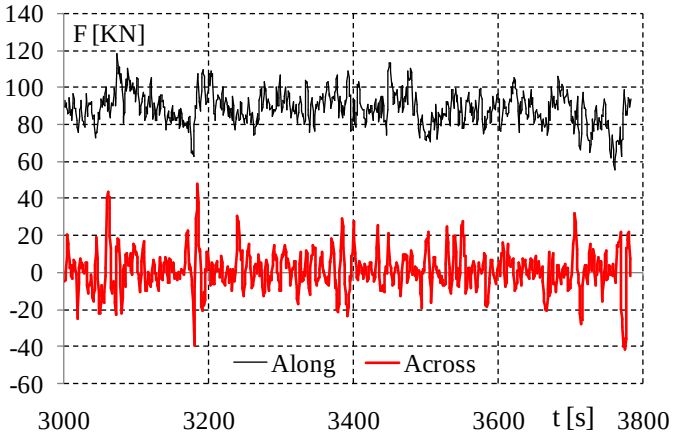
Vortex shedding phenomenon



Experimental results (wind tunnel testing)

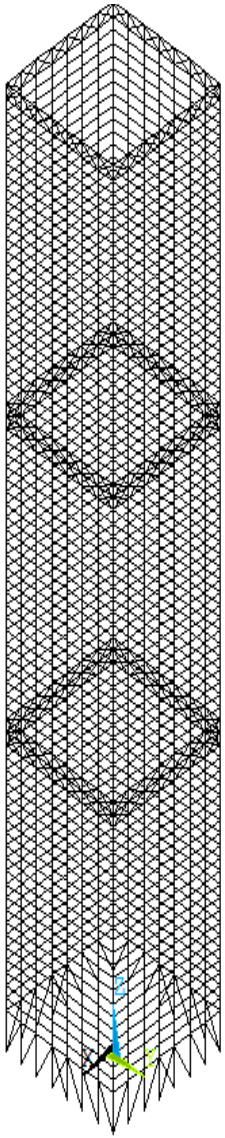


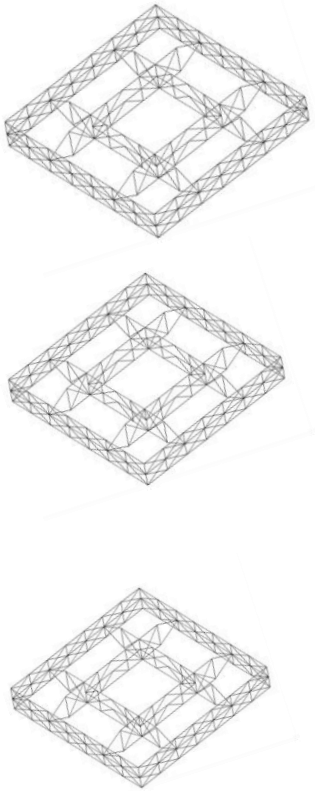




Case study of a typical tall building subject to vortex shedding

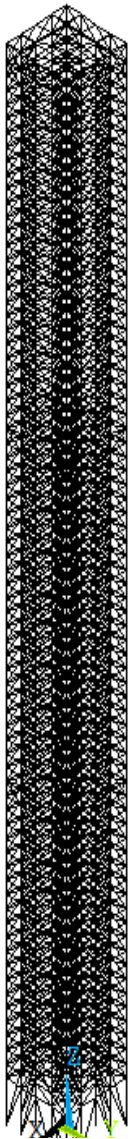
Perimetric frame system



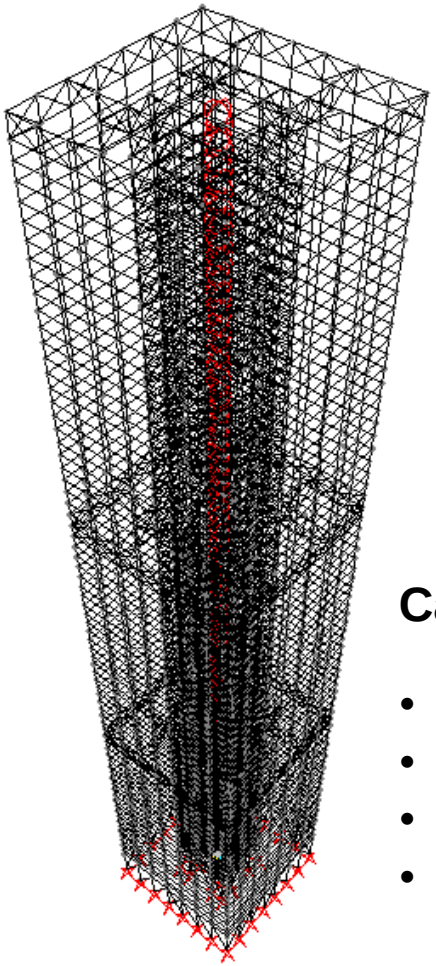


Bracing system (outriggers)

Central core



Complete FE model



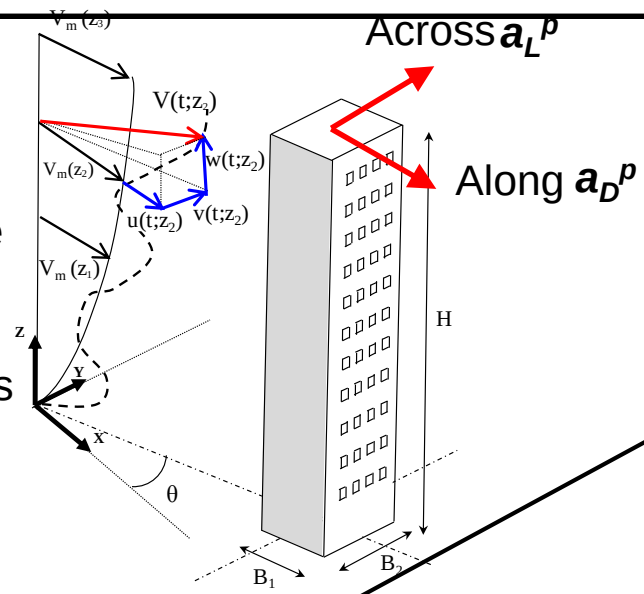
Ciampoli/Petrini, *PEM* (2011)

Case study- Structure

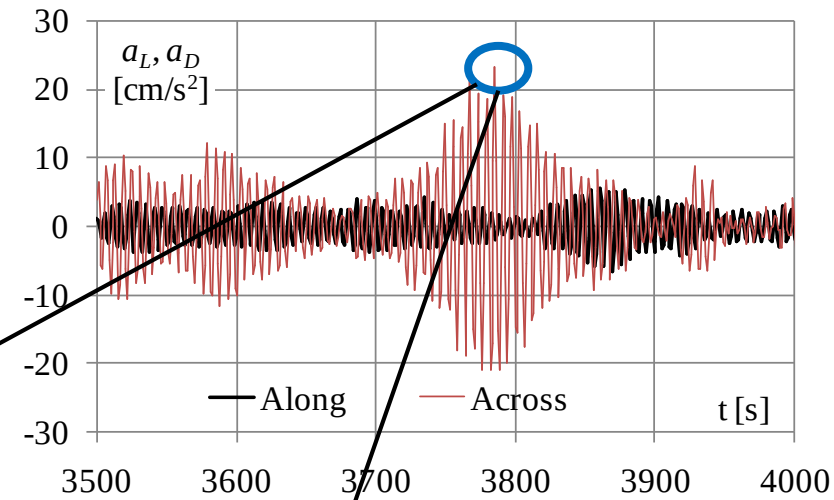
- 74 floors
- Height H=305m
- Footprint $B_1=B_2=50$m
- Total mass= 92500 tons

Case study of a typical tall building subject to vortex shedding

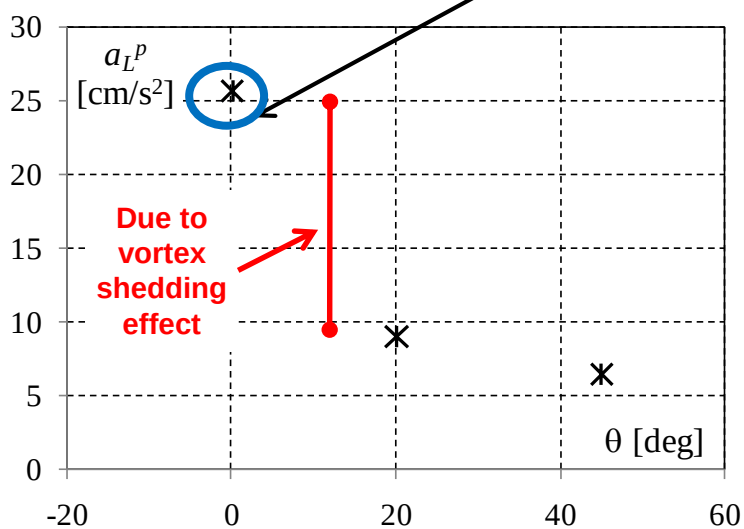
For buildings having H/B>3 the across-wind direction is dominant in terms of accelerations



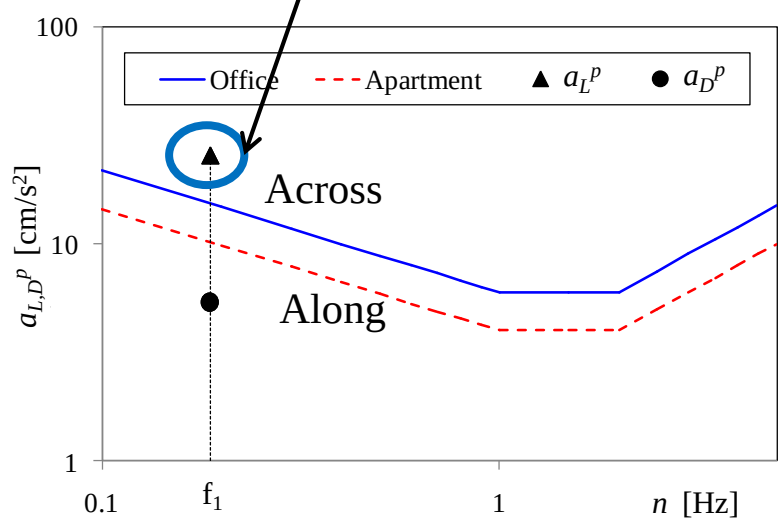
Structural response – accelerations



Peak response accelerations



Comfort evaluation



Considered vortex shedding input spectra

Petrini/Giaralis (2016)

$$\frac{nS_{ffy}(n)}{\sigma_j^2} = A\frac{H(C_1)\overline{n}^2}{\left(1-\overline{n}^2\right)^2+C_1\overline{n}^2} + ($$

Being

$$H(C_1) = 0.179C_1 + 0.65\sqrt{C_1}$$

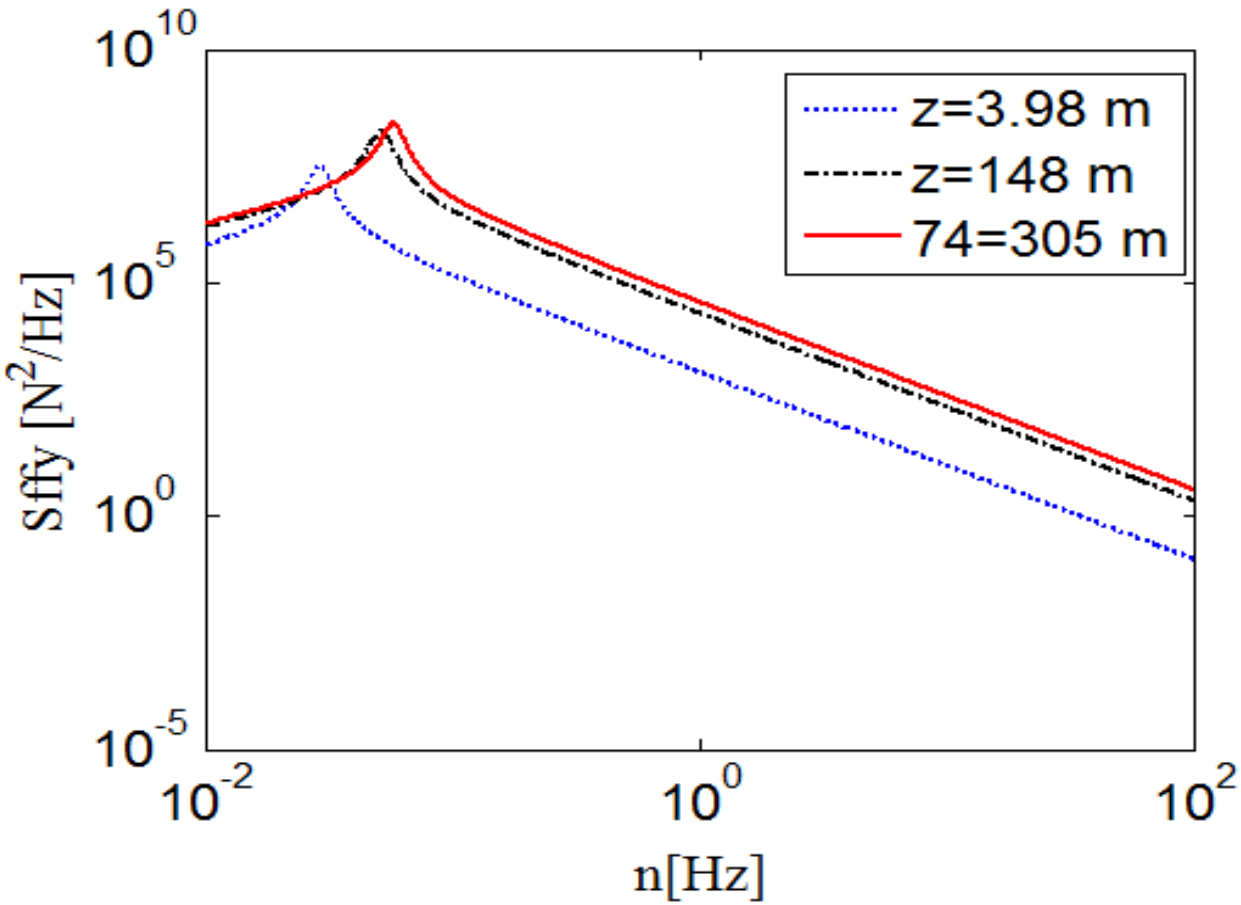
$$C_1 = \left[0.47\left(\frac{D}{B}\right)^{2.8} - 0.52\left(\frac{D}{B}\right)^{1.}\right.$$

$$A = \left(H_{tot}/\sqrt{S}\right)\left[-0.118\left(\frac{D}{B}\right)^2\right.$$

$$\sigma_j = \frac{1}{2}U\left(z_j\right)^2 B\overline{C}_L$$

And

$$\overline{C}_L = 0.045\left(\frac{D}{B}\right)^3 - 0.335\left(\frac{D}{B}\right)^2 + 0.868\left(\frac{D}{B}\right) - 0.174$$



Structural analysis in the frequency domain

Petrini/Giaralis (2016)

Input/output relationship from random vibrations theory

$$\mathbf{S}_{QQ}(\omega) = \mathbf{B}(\omega)^{*}\,\mathbf{S}_{FFy}(\omega)\,\mathbf{B}(\omega)$$

$$\mathbf{S}_{\ddot{Q}\ddot{Q}}(\omega) = \omega^{4}\mathbf{S}_{QQ}(\omega)$$

Where

$$\mathbf{S}_{FFy}(\omega) = 2\pi\mathbf{S}_{FFy}(n)$$

And

$$\mathbf{B}(\omega) = \left[\mathbf{K} - \omega^{2}\mathbf{M} + i\omega\mathbf{C}\right]^{-1}$$

Response variances

$$\sigma^{2}_{Q_j}(\omega) = \int_{0}^{\omega_{\max}} S_{Q_jQ_j}(\omega)\,d\omega$$

$$\sigma^{2}_{\ddot{Q}_j}(\omega) = \int_{0}^{\omega_{\max}} S_{\ddot{Q}_j\ddot{Q}_j}(\omega)\,d\omega$$

Peak responses

$$Q_j^{p} = g\sqrt{\sigma^{2}_{Q_j}}$$

$$\ddot{Q}_j^{p} = g\sqrt{\sigma^{2}_{\ddot{Q}_j}}$$

$$g = \sqrt{2ln(\eta T_{wind})} + \frac{0.577}{\sqrt{2ln(\eta T_{wind})}}$$

Model reduction from detailed finite element model (FEM)

1) A diagonal mass matrix is assumed corresponding to a lumped mass 74 DOF system populated by the floor masses

2) Natural frequencies (eigenvalues) and mode shapes (eigenvectors) corresponding to the 74 lateral dominantly translational are obtained from the detailed FEM model

Petrini/Giaralis (2016)

3) Derivation of a full stiffness matrix

$$\left[\mathbf{K}_{(0)} - \omega^2_{(\mathrm{FEM})_j}\mathbf{M}_{(0)}\right]\boldsymbol{\varphi}_{(\mathrm{FEM})_j} = 0$$

with $j=1,2,\ldots,74$

3) Derivation of a full damping matrix

$$\mathbf{C}_{(0)} = \left(\boldsymbol{\Phi}^T\right)^{-1}\mathbf{C}_{\mathrm{mod}}\left(\boldsymbol{\Phi}\right)^{-1}$$

$$c_j = 2\omega_j\xi_j\left(\boldsymbol{\varphi}_j{}^T\mathbf{M}_{(0)}\boldsymbol{\varphi}_j\right)$$

Mode j	Assumed damping ratio
1-3	2%
4-6	4%
7-10	6%
11-20	9%
21-40	12%
41-60	15%
>60	18%

Case study of a typical tall building subject to vortex shedding

Independent parameters

β= 0:1

μ= 0.3%; 0.5%; 0.8%; 1%; 1.5%

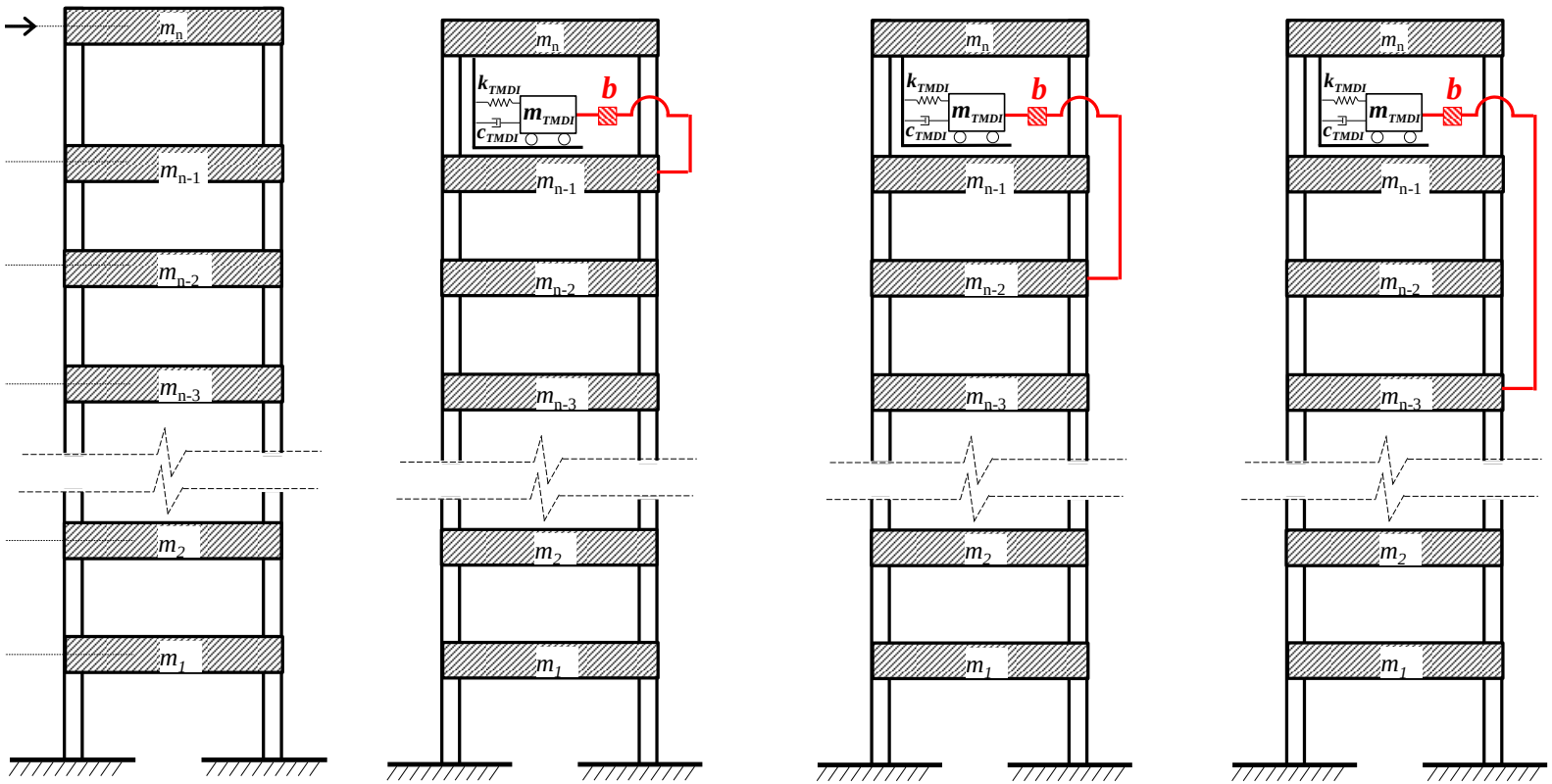
damping ratio

$$\zeta_{TMDI} = \frac{(\beta+\mu)\left[1+0.75(\beta+\mu)\right]}{4(1+\mu+\beta)\left[1+0.5(\beta+\mu)\right]}$$

frequency ratio:

$$\upsilon = \frac{\sqrt{1+0.5(\mu+\beta)}}{1+\mu+\beta}$$

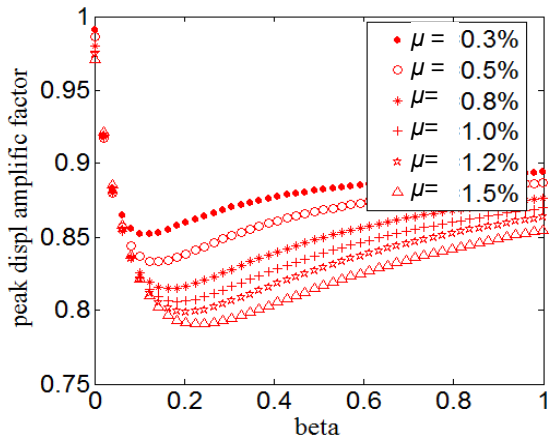
TMDI Connectivities -1 -2 -3

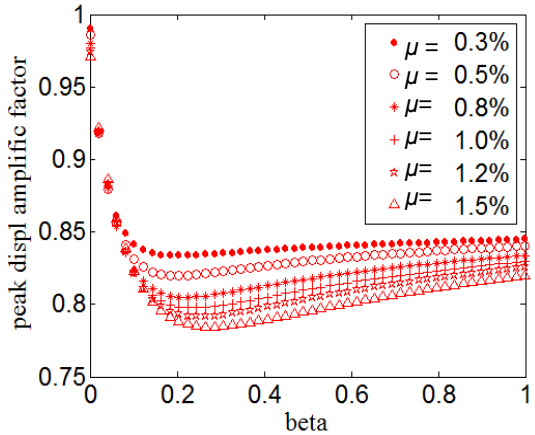


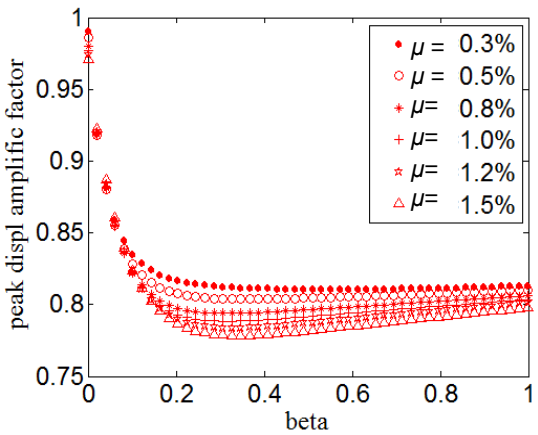
Case study of a typical tall building subject to vortex shedding

Petrini/Giaralis (2016)

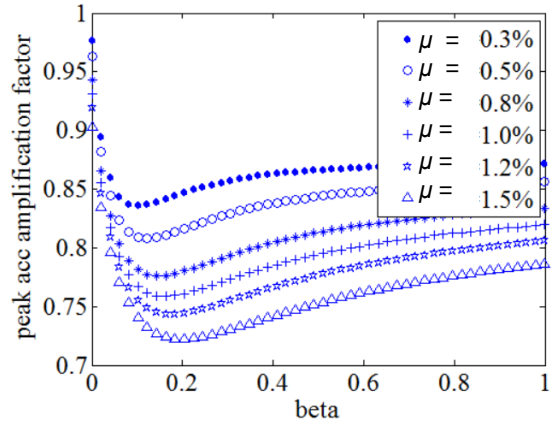
Top floor peak displacement

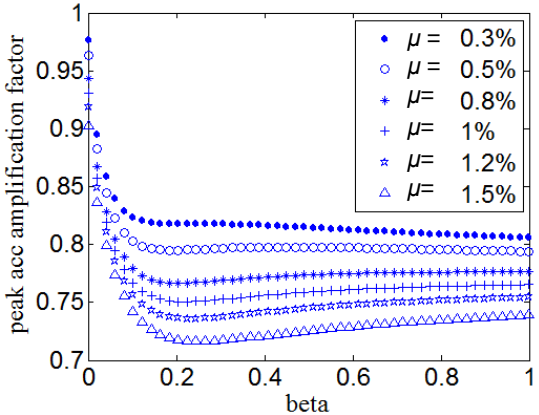


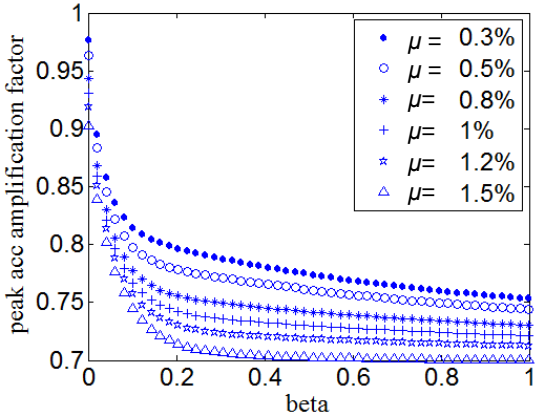




Top floor peak acceleration

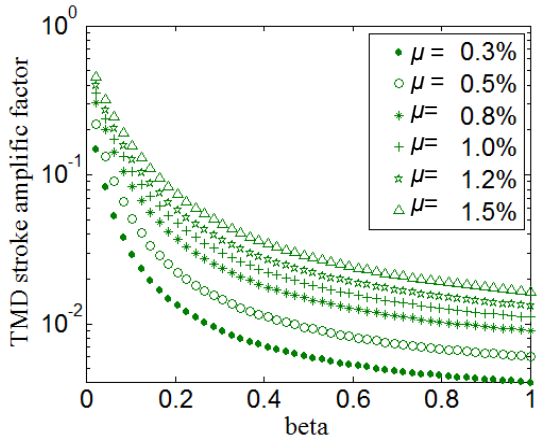


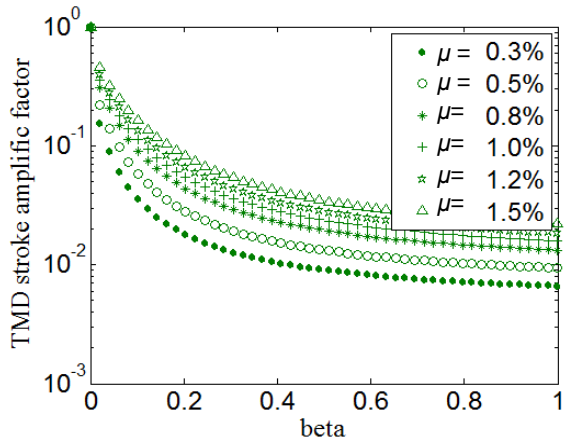


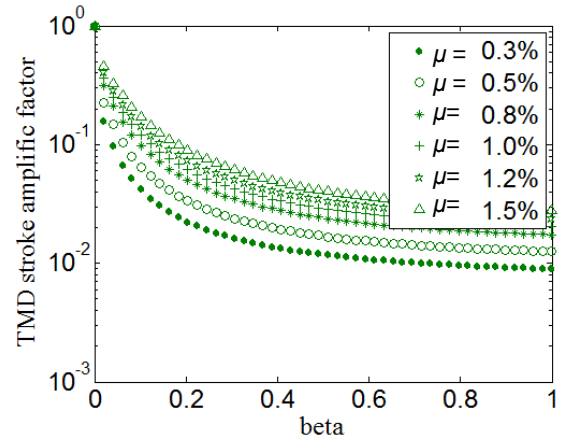


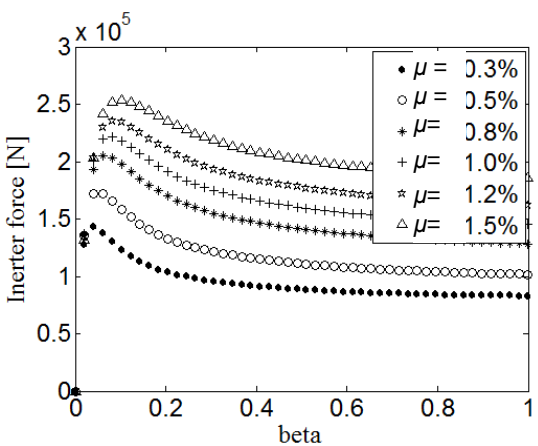
-1 **-2** **-3**

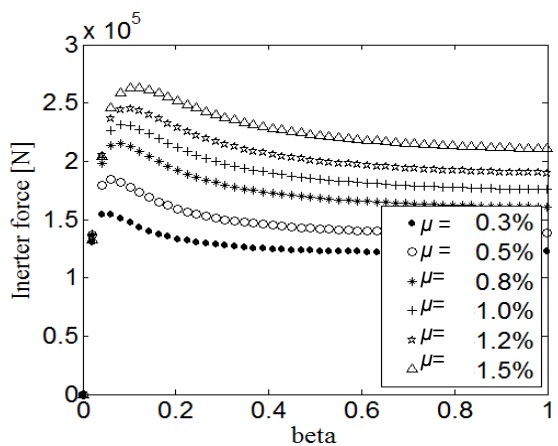
TMDI connectivity

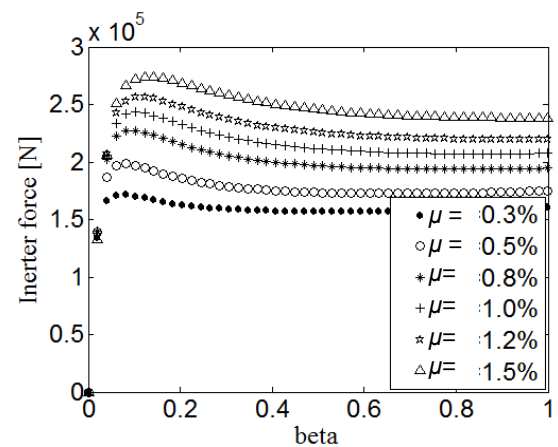












Case study of a typical tall building subject to vortex shedding

Petrini/Giaralis (2016)

Peak TMDI Stroke

Peak inerter forces

-1 **-2** **-3**

TMDI connectivity

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Case study of a typical tall building subject to vortex shedding

Sub-optimal inertance values and mass amplification factor

Conn type	Response type	Parameter	Sub-optimal values of β					
			μ=0.3%	μ=0.5%	μ=0.8%	μ=1.0%	μ=1.2%	μ=1.5%
-1	Displ	β	*0.1224*	0.1429	0.1633	0.1837	0.2041	0.2245
-2	Displ	β	0.2041	0.2041	0.2245	0.2449	0.2449	0.2857
-3	Displ	β	0.5541	0.3571	0.3163	0.3163	0.3265	0.3367

Conn type	Response type	Parameter	Response amplification factors at sub-optimal values of β					
			μ=0.3%	μ=0.5%	μ=0.8%	μ=1.0%	μ=1.2%	μ=1.5%
-1	Displ	ampl factor	*0.8523*	0.8333	0.8149	0.8061	0.7991	0.7909
-2	Displ	ampl factor	0.8339	0.8198	0.8049	0.7976	0.7971	0.7847
-3	Displ	ampl factor	0.8108	0.8038	0.7938	0.7884	0.7839	0.7785

The same response can be equivalently obtained by a classical TMD with μ=3.12%, and the same η, ξ values

Mass/weight reduction by about 10 times

Petrini/Giaralis (2016)

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EST 1894

Major concluding remarks

- The TMDI exploits the mass amplification effect of the inerter to:

  - improve the classical TMD performance for a fixed oscillating additional mass, or to

  - "replace" part of the TMD vibrating mass by achieving an overall lighter passive control solution

This is an important consideration, especially for earthquake engineering applications and for tall buildings under wind excitations

- The TMDI controls more than one modes of vibration which may or may not be accounted for in the design

This is an important consideration in suppressing floor accelerations in earthquake and wind excited buildings

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THANK YOU FOR YOUR ATTENTION

- Marian Laurentiu City University London studentship (2012-2015)

  PhD thesis (viva 12/2016):
  The tuned mass damper inerter for vibration suppression and energy harvesting in dynamically excited structural systems

- EPSRC

Big Pitch- Bright Ideas award (10/2014)

Multi-objective performance-based design of tall buildings using energy harvesting enabled tuned mass-damper-inerter (TMDI) devices

EPSRC Pioneering research and skills

ideas FACTORY

Grant Ref: EP/M017621/1

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- Assoc. Prof. Alexandros Taflanidis, University of Notre Dame, USA