Optimal Control of Formula One Car Energy Recovery Systems

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with

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• Nominal Car Performance & Setting the Scene
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Control of Energy Management:
- One sensor is connected to measure effective energy into and out of the Energy Store.
- One Sensor is connected to measure effective energy into and out of the MGU-H.
- The UC-U sensor may only contain energy. This will be verified by inspection.
Thermal Energy Recovery
2014 Power Train I

- Thermal Energy Recovery system
- Kinetic Energy Recovery System
- Lithium Ion Battery
- Turbo-compounded IC engine
2014 Power Train II

1. 100 kg of fuel per race;
2. the fuel mass flow limit 100 kg/hour.
Track Modelling
Track as a Ribbon

Frenet-Serret apparatus
Lemniscate of Bernoulli
# Track Model I

<table>
<thead>
<tr>
<th>Corner</th>
<th>Distance (m)</th>
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<td>SF</td>
<td>7004</td>
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Track Model II

Geodesic curvature

Elevation
\[ \dot{s} = \frac{u \cos \xi - v \sin \xi}{1 - n \Omega z} \]

\[ w = n \omega_x. \]

\[ \dot{n} = u \sin \xi + v \cos \xi. \]
Car Modelling
Car Modelling

- Newtonian mechanics;
- Closed kinematic suspension loops;
- Meta modelling;
- Magic Formula-type tyre;
- Aerodynamic maps;

Aerodynamics are for people who can't build engines.

Enzo Ferrari
Car Modelling

- Newtonian mechanics;
- Closed kinematic suspension loops;
- Magic Formula-type tyre;
- Speed-dependent aerodynamic maps;

Aerodynamics are for people who can't build engines.

Enzo Ferrari
Suspension System
Suspension System
Car Dynamics
Partial EOM

\[ \ddot{u} = (\nu + h\bar{\omega}_x)\bar{\omega}_z - n\omega_x\bar{\omega}_y + g (\sin\xi \sin\phi \cos\mu - \cos\xi \sin\mu) + F_x/M \]

\[ \ddot{v} = n\omega_x\bar{\omega}_x - (u - h\bar{\omega}_y)\bar{\omega}_z - g (\sin\xi \sin\mu + \cos\xi \sin\phi \cos\mu) + F_y/M \]

\[ \ddot{\omega}_z = ((I_x - I_y)\bar{\omega}_x\bar{\omega}_y + a (\cos\delta (F_{fr y} + F_{fl y}) + \sin\delta (F_{fr x} + F_{fl x})) + w_f (\sin\delta F_{fr y} - \cos\delta F_{fr x}) - w_r F_{rr x} + w_f (\cos\delta F_{fl x} - \sin\delta F_{fl y}) + w_r F_{rl x} - b(F_{rr y} + F_{rl y})) / I_z, \]
Tyre Model
## Aerodynamic Maps

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Optimal Control
Optimal Control Problem

\[
\frac{dx}{d\tau} = f(x(\tau), u(\tau), \tau), \quad x(-1) = x_0,
\]

**Mayer-Pontryagin Cost**

\[
\Phi(x(1))
\]

Control Hamiltonian

\[
\mathcal{H}(x, \lambda, u, \tau) = \lambda^T f
\]

**PMP**

\[
u^* = \arg \min_u \mathcal{H}(x, \lambda, u, \tau)
\]

**TPBVP**

\[
\begin{cases}
\dot{x} = \nabla_\lambda \mathcal{H} & x(-1) = x_0 \\
\dot{\lambda} = -\nabla_x \mathcal{H} & \lambda(1) = \nabla \Phi(x(1))
\end{cases}
\]
Numerical Optimal control

\[ \min_U \Phi(X_N) \]

\[ D_{1:N} X_{1:N} = F(X, U) - D_0 x_0; \]

\[ D_{1:N}^T \Lambda = \nabla_X < \Lambda, F(X, U) > + e_N \nabla \Phi(X_n) \]

\[ 0 = \nabla_U < \Lambda, F(X, U) >, \]

Discrete Mayer cost

Discrete dynamics

NLP Karush-Kuhn-Tucker (KKT) optimality conditions
Practicalities

- Scaling important;
- Non-smooth features must be approximated;
- Singular arcs regulated

\[ J = \int_{0}^{T} (1 + \sum_{i=1}^{m} \epsilon_i u_i^2) \, dt \]

- Direct methods with Gaussian quadrature integration schemes method of choice;
- Computations use GPOPS-II, IPOpt and ma57.
Non-smooth Features

\[ \max(x, 0) \approx \frac{x + \sqrt{x^2 + \epsilon}}{2} \quad \text{and} \quad \min(x, 0) \approx -\frac{-x + \sqrt{x^2 + \epsilon}}{2}, \]

\[
\frac{\partial}{\partial x} \left( \frac{1}{2} \left( x + \sqrt{x^2 + \epsilon} \right) \right) = \frac{1}{2} \left( \frac{x}{\sqrt{x^2 + \epsilon}} + 1 \right)
\]

\[ |x| \approx \sqrt{x^2 + \epsilon} \]
Optimal Car Performance
Refined Mesh
Speed Profile of Nominal Car
The Racing Line; corners 10-16
Optimal Lap of Barcelona
Conventional 2013 KERS
Kinetic Energy Recovery

• 2013 kinetic energy recovery systems (KERS) facilitate the capture of kinetic energy that derives from braking;
• Recovered energy can be stored for later deployment in propelling the car;
• Short-term power boost to be used under over taking.
**Conventional KERS Constraints**

$P_{max}^{kers} = \pm 60 \text{ kW}$

Engine only

\[ \begin{align*}
P_{IC}^{max} + P_{kers} - P_m & \geq 0 \\
P_{kers} - P_m H(-P_m) & \geq 0
\end{align*} \]

Cannot use engine to recharge batteries.
Power Usage

Bang-Bang controls
Energy Usage

Energy Quota 400kJ
2014 Power Train
2014 Power Train Features

• Controls: steering, brakes, fuelling, waste gate (MGU-H), MGU-K;
• The car's energy store (ES) up to 4 MJ;
• ES-2-MGU-K up to 4MJ per lap;
• MGU-K-2-ES up to 2MJ per lap;
• MGU-K restricted to ±120 kW.
Simulation Track - Spa

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Energy Usage (racing)
Power Usage (racing)
Speed Difference and Long Gravity

Up Hill

Down hill
Power Unit Maps
Power Unit Maps

Diagram showing the relationships between different parameters:

- \( P_D \) and \( P_m \) connected to \( W_g \)
- \( P_m \) connected to \( \dot{F} \)
- \( P_k \) and \( \omega_m \) connected to each other
Power Usage (Racing)
Optimised 2D & 3D maps
Conclusions & Future Work I

- Optimal control of thermal-electric power trains accomplished;
- Like-for-Like performance with 2/3 fuel consumption;
- Three-dimensionality can be important and can be studied;
- Power train maps can be parameterised and optimised;
- Fast dynamics relating to aero-suspension treated quasi-statically;
- Further progress being made in the optimal control of high-fidelity models...

Thanks for Listening