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Abstract

We argue that the forward discount puzzle is primarily a statistical phenomenon and that statistical rejections of Uncovered Interest Parity do not necessarily constitute valid rejections of market efficiency. We find by using a Taylor expansion a theoretical negative bias in existing regressions of UIP. We propose two alternative tests for market efficiency, one of which is designed to measure the degree of market inefficiency. Our results from these tests indicate that for all four of the bilateral dollar parities studied the foreign exchange market is efficient despite decisive clear rejections of UIP using the conventional regression approach.

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1. Introduction

The forward discount puzzle has eluded an acceptable solution for the best part of three decades. In their recent study, Sarno *et al* (2006) state that "the forward bias puzzle has not been convincingly explained and continues to baffle the international finance profession." The forward discount puzzle is also known as the Uncovered Interest Parity (UIP) puzzle. In a nutshell, the forward discount puzzle is that according to UIP condition the currency which has the high interest rate is expected to depreciate. However, when tests of the UIP equation are conducted by regressing the change in the log of the exchange rate against the forward discount, the regressions have almost universally shown a negative coefficient which is usually statistically significant, rather than a value of unity. Indeed, as Froot and Thaler (1990) show, the regression coefficients are usually closer to minus unity than the expected value of plus unity. Taken at face value, the regression estimates suggest that currencies at a forward discount (that is, the high interest rate currency) appreciate rather than depreciate as one would expect in an efficient foreign exchange market. As such, the conventional regression results can be interpreted as indicating a significant degree of foreign exchange market inefficiency.

There are numerous studies that document the failure of the change in the exchange rate to match the predicted depreciation/appreciation inherent in the forward discount/premium. These include *inter alia* Longworth (1981), Boothe and Longworth (1986), Hodrick (1987), Froot and Thaler (1990), Engel (1996), Baillie and Bollerslev (2000), Maynard and Phillips (2001) and Sarno *et al* (2006). The supposed failure of UIP has led to a plethora of studies that investigate the potential reasons for the failure of

foreign exchange market efficiency. One possibility is the existence of a risk premium (which may be time varying) but this idea is rejected by Cumby (1988) and Frankel and Froot (1989). Another explanation, see for example, Taylor (1989) and Cavaglia *et al* (1994) argues that the forward bias is the result of either rational or irrational bubbles. More recently, there has been a questioning of statistical validity of the results obtained by using conventional regression techniques, Baillie and Bollerslev (2000) argue that there may be biases in the conventional regression estimates which might make the rejections of UIP a statistical phenomenon. While Sarno *et al* (2006) show that some progress can be made by modelling deviations from UIP as a nonlinear process.

In this paper, we argue that the conventional regression estimates of UIP that are obtained by regressing the change in the log of the exchange rate against the forward discount/premium are seriously misleading in suggesting a significant degree of foreign exchange market inefficiency. The regression estimates contain a negative bias given by the variance of the ratio of the future spot rate to the forward rate. By using a Taylor expansion we find the exact relationship between the difference in the log of the exchange rate and the forward discount/premium. We show that the inclusion of a set of relevant variables accounting for deviations of the future spot exchange rate in levels from its forward price enable us to correctly test for foreign exchange market efficiency. The presence of these extra terms prevents the conventional UIP condition from holding even in an efficient market unless the ratio of these two sequences (the future spot exchange rate and forward price both in level form) is *systematically* very close to one. We propose two tests, one of which tests for market efficiency and the other which is a

more general test that measures the degree of inefficiency should any inefficiency exist. Our results are somewhat startling in showing that the foreign exchange market is shown to be efficient for all four bilateral dollar parities despite decisive rejections of UIP in all four cases using the conventional regression approach.

The paper is set out as follows, in section 2 we look at the conventional regression set up that has resulted in the forward discount puzzle and briefly review some of the possible solutions that have been put forward in the literature in an attempt to resolve the puzzle. Section 3 sets out our theoretical framework for our proposed resolution of the forward discount puzzle. In section 4 we study the application of our tests to exchange rate data of four major currencies *vis-a-vis* the US dollar. The results from our methodology are compared to those obtained by using the conventional regression approach. The results we believe represent a possible resolution of the forward discount puzzle. We finish the study in Section 5 by concluding that the forward discount puzzle is primarily a statistical phenomenon and that the foreign exchange market is efficient. We also outline possible implications for future research.

2. Conventional Tests of the UIP Condition

The econometric testing of the UIP condition and its link to the forward discount puzzle is usually derived using the following steps. It is assumed that the covered interest rate parity condition [(1)] which is an arbitrage condition used to calculate the forward rate holds as depicted by equation (1):

$$F_t = \frac{(1+r_t^*)S_t}{(1+r_r)} \qquad (1)$$

where S_t is the spot exchange rate at time t defined as units of foreign currency per unit of domestic currency (US dollars), F_t is the forward exchange rate at time t, r_t^* is the foreign interest rate and r_t the domestic (US) interest rate[2]. Taking logs of equation (1) means that CIP formula can be approximated by equation (2):

$$f_t - s_t = r_t^* - r_t \tag{2}$$

with f_t and s_t the logs of the forward exchange rate and the spot exchange rate at time t respectively.

The efficient market hypothesis (EMH) will hold if we assume rational expectations, risk neutrality, no taxes on capital transfers and perfect capital mobility. Under these conditions then the expected future spot rate will be equal to the forward exchange rate as given by equation (3):

$$E[S_{t+1}] = F_t \tag{3}$$

The Uncovered Interest Parity (UIP) condition states that the interest rate differential is, on average, equal to the expected rate of change in the natural log of the exchange rate as given by equation (4):

$$E[s_{t+1}] - s_t = r_t^* - r_t \tag{4}$$

where $E[s_{t+1}]-s_t$ is the expected rate of depreciation of the currency. If we impose rational expectations we have:

$$s_{t+1} = E[s_{t+1}] + \varepsilon_{t+1} \tag{5}$$

where s_{t+1} is the natural log of the spot exchange rate at time t+1 and ε_{t+1} is a random error term with zero mean and normal distribution. Substituting equation (4) into

equation (5) and using the approximation given by equation (2) we obtain a testable form of UIP given by equation (6):

$$s_{t+1} - s_t = (f_t - s_t) + \varepsilon_{t+1} \tag{6}$$

The conventional regression equivalent test of UIP is given by equation (7):

$$\Delta s_{t+1} = \alpha + \beta(f_t - s_t) + \varepsilon_{t+1} \tag{7}$$

where Δs_{t+1} denotes $(s_{+1} - s_t)$ the change in the natural log of the exchange rate. If UIP holds then the beta coefficient in such regressions is unity, the intercept is zero and the error term white noise. Regression based tests of UIP using equation (7) have performed very poorly – as can also be seen in results which we report in table 1. The estimates of the slope parameter (β) in the standard regression equation are typically negative and usually significantly so, as is the case for the four parities that we study. These regression based results have resulted in the forward bias puzzle, it appears to be the case that currencies at a forward discount on average appreciate rather than depreciate as required by the UIP condition.

There have been a variety of attempts to improve upon the standard regression model. Under the umbrella of single equation regressions, there are two types of tests, one using non-overlapping data and the other using overlapping data. The former was introduced by Fama (1984), who shows that Ordinary Least Squares (OLS) estimators perform well, while the case of overlapping data is covered in Hansen (1982) and represented by the use of Generalized Method of Moments (GMM) estimators. Wang and Jones (2002) summarize the results of tests on UIP based on these methods and they find that the rejection of the UIP hypothesis remains overwhelming. The VAR approach to

modelling foreign exchange rate behaviour is claimed to be more efficient than single regression equations in the study of the UIP hypothesis and is investigated by Hakkio (1981), Baillie (1989) and MacDonald and Torrance (1989) amongst others. In this context, the joint process driving the forward premium and the increment of the spot exchange rate is specified. A typical model is as follows:

$$\Delta s_{t+1} = \sum_{i=1}^{m} \alpha_{1i} \Delta s_{t+1-i} + \sum_{j=1}^{m} \beta_{1j} \Delta f_{t+1-j} + \varepsilon_{1,t+1} \tag{8}$$

$$\Delta f_{t+1} = \sum_{i=1}^{m} \alpha_{2i} \Delta s_{t+1-i} + \sum_{j=1}^{m} \beta_{2j} \Delta f_{t+1-j} + \varepsilon_{2,t+1} \tag{9}$$

with $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ the error terms in the equations and satisfying $E[\varepsilon_{i,t}] = 0$ and $E[\varepsilon_{i,t}\varepsilon_{j,t-k}] = 0$ for *i, j* = 1, 2 and $k \neq 0$. In this framework, the UIP hypothesis is tested by imposing constraints on the VAR coefficients matrix. However, the use of these models has not changed the results on tests of the relationship between the forward exchange rate and the spot rate in the next period. Furthermore, the inclusion of more sophisticated econometric models measuring long-run relationships and testing for cointegration between the variables does not vary the sign of the slope estimates and potentially adds more sources of error in testing the UIP condition. Wang and Jones (2002) find that the basic VAR equations applied for UIP are only able to test whether the necessary conditions hold and as such have nothing to say about efficiency and rationality. They propose an extension of the previous bivariate model to describe the dynamics of the exchange rate in levels, but their results on UIP are as disappointing as in the rest of the literature.

A significant clue concerning the forward discount puzzle is provided in the paper of Baillie and Bollerslev (2000). The authors note that while daily exchange rate returns exhibit pronounced volatility clustering, the time series corresponding to the forward premium hardly shows only minor departures from its expected value. They provide evidence that the failure of UIP may be a statistical phenomenon that occurs because of the very persistent autocorrelation in the forward premium implying that the conventional regression equation may not be balanced. In other words, the order of integration of the dependent and explanatory variables is not the same. In this situation Baillie and Bollerslev (1994) claim that a test of the UIP condition should be based on estimating the relationship:

$$\Delta s_{t+1} = \alpha + \beta[1-(1-L)^{1-d}](1-L)^{d}\phi(L)(f_t - s_t) + \varepsilon_{t+1} \qquad (10)$$

and testing whether $\alpha=0$, $\beta=1$ and ε_{t+1} is serially uncorrelated. In equation (10) L stands for the lag operator and d is the fractional order of integration with $0<d<1$.

We also argue that the conventional regression test is likely to reject the UIP condition due to substantially different characteristics in the statistical properties exhibited by the time series on the two sides of the standard regression equation. This can readily be seen by inspection of our figure 1 which reveals much higher levels of volatility of the change in the log of the exchange rate compared to the volatility of the forward premium/discount[(3)]. Indeed, given the very large differences in volatility it is surprising that any real credibility can be attached to the conventional regression results.

3 An alternative regression equation to test for UIP and market efficiency

In an efficient foreign exchange market we would expect $E[S_{t+1}] = F_t$. However, the UIP condition tested in the literature and given by equation (7) is equivalent to assuming $E[s_{t+1}] = f_t$. In general, however, these two equations are not equivalent. We now show the relation between these two market conditions. By Jensen's inequality, and writing $S_{t+1} = e^{s_{t+1}}$ and $F_t = e^{f_t}$ we can see that if UIP holds - as given by equation (6), ignoring the error term, then we have:

$$E[S_{t+1}] = E[e^{s_{t+1}}] \geq e^{E[s_{t+1}]} = e^{f_t} = F_t . \tag{11}$$

On the other hand, if the foreign exchange market is efficient the relation between the variables in logs is given by:

$$E[s_{t+1}] \leq f_t \ . \tag{12}$$

These two conditions are equivalent only for very small departures of the ratio $\frac{S_{t+1}}{F_t}$ from unity, where the linear approximation holds.

The first contribution in this section is to find the exact relationship between the expected value of the change in the log of the exchange rates and the interest rates differentials in an efficient market, and to see if UIP condition and market efficiency are equivalent conditions.

Taking the nth order Taylor approximation[(4)] of s_{t+1} about $E[S_{t+1}]$, we have:

$$\begin{aligned} s_{t+1} = \ln E[S_{t+1}] &+ \frac{1}{E[S_{t+1}]}(S_{t+1} - E[S_{t+1}]) - \frac{1}{2E^2[S_{t+1}]}(S_{t+1} - E[S_{t+1}])^2 \\ &+ \frac{1}{3E^3[S_{t+1}]}(S_{t+1} - E[S_{t+1}])^3 + \ldots + \frac{1}{nE^n[S_{t+1}]}(S_{t+1} - E[S_{t+1}])^n \end{aligned} \tag{13}$$

For n=3 the expected value of the log of the exchange rate is as follows:

$$E[s_{t+1}] = \ln E[S_{t+1}] - \frac{1}{2E^2[S_{t+1}]} Var(S_{t+1}) + \frac{1}{3E^3[S_{t+1}]} Skew(S_{t+1}) \tag{14}$$

where $Var(S_{t+1})$ is the conditional variance of S_{t+1} and $Skew(S_{t+1})$ the conditional skewness of S_{t+1}. Thus, from equation (13) if the market is efficient the difference of the log exchange rate is given by the following:

$$\Delta s_{t+1} = (f_t - s_t) + [(\frac{S_{t+1}}{F_t}) - 1] - \frac{1}{2}[(\frac{S_{t+1}}{F_t}) - 1]^2 + \frac{1}{3}[(\frac{S_{t+1}}{F_t}) - 1]^3 \tag{15}$$

and the expected rate of depreciation is:

$$E[\Delta s_{t+1}] = (f_t - s_t) - \frac{1}{2} Var[\frac{S_{t+1}}{F_t}] + \frac{1}{3} Skew[\frac{S_{t+1}}{F_t}] \tag{16}$$

where $Var(S_{t+1}/F_t)$ and $Skew[S_{t+1}/F_t]$ are the conditional variance and conditional skewness of the ratio S_{t+1}/F_t. If the conditional distribution of S_{t+1}/F_t is symmetric $Skew[S_{t+1}/F_t]=0$ and hence ignoring the skew parameter, equation (16) can be re-written when combined with equation (2) as either:

$$E[\Delta s_{t+1}] = (f_t - s_t) - \frac{1}{2} Var[\frac{S_{t+1}}{F_t}] \tag{17}$$

or

$$E[\Delta s_{t+1}] = (r_t^* - r_t) - \frac{1}{2} Var[\frac{S_{t+1}}{F_t}] \tag{18}$$

given that CIP holds and that the forward discount is closely approximated by the difference of interest rates.

A result similar in spirit to that of equations (17) and (18) is found in the context of two open economies with one domestic risk-free asset and one foreign risky asset, and where investors' preferences are modelled by quadratic utility functions. In this setting, the result is an equilibrium market condition explained as a result of a risk premium on the foreign currency. However, it is clear from the previous arguments that this relationship holds even in an efficient market. The term $-1/2[(\frac{S_{t+1}}{F_t})-1]^2$ does not vanish when taking expectations. The lower panels of figure 2 show that the term $-1/2[(\frac{S_{t+1}}{F_t})-1]^2$ for the exchange rates under study is nonetheless very close to zero and consequently, at least for the data we analyse, the standard UIP formula and market efficiency are equivalent conditions [(5)]. The market will be efficient and UIP hold if:

$$E[\Delta s_{t+1}] = (r_t^* - r_t) \tag{19}$$

or

$$E[\Delta s_{t+1}] = (f_t - s_t) \tag{20}$$

However, as mentioned before, the standard regression equation (7) used to test equations (19) or (20) has shown disappointing results pointing towards significant inefficiency in the foreign exchange market. This is in large part due to the differences in magnitude between the two relevant sequences. While Δs_{t+1} exhibits significant levels of volatility the forward discount/premium shows markedly less volatility, for example, in the case of the pound the variance of the left hand side of equation (7) is 223 times that of the right hand side! (see footnote 3 for the other parities). It is easy to observe from figure

1 that the error term in equation (7) does not differ much from simply studying Δs_{t+1}, consequently the results from a regression like equation (7) are likely to be spurious.

The next contribution of the section is to develop a simple and intuitive technique to test the UIP condition and the extent of market efficiency/inefficiency in the foreign exchange market. From equation (15) and assuming no skewness and negligible value of the volatility term we have:

$$\Delta s_{t+1} - (f_t - s_t) = [(\frac{S_{t+1}}{F_t}) - 1] \tag{21}$$

Rather than studying the left hand term we propose to study the right hand term ratio as a test of market efficiency. Note that equation (21) is only an approximation of the exact relationship given by equation (15), however, an analysis of the lower panels in figure 2 shows clear evidence of the accuracy of the approximation. If the foreign exchange market is efficient then $[(\frac{S_{t+1}}{F_t}) - 1] = 0$ and a useful regression test would be:

$$[(\frac{S_{t+1}}{F_t}) - 1] = \alpha + \varepsilon_{t+1} \tag{22}$$

with ε_{t+1} making allowance for the conditional heteroskedasticity. The efficient market hypothesis is that α = 0. Note that a regression of equation (22) is similar in spirit to the first econometric models proposed to test market efficiency and given by $S_{t+1} = \alpha + \beta F_t + \varepsilon_{t+1}$, see for example, Levich (1978) and Frenkel (1982). However, in contrast to that model, the dependent variable in equation (22) is stationary. In particular, for the four monthly series studied, we reject the null hypothesis of a unit root in the

sequence $(\frac{S_{t+1}}{F_t})-1$. In the upper panels of figure 2 we plot the sequence $(\frac{S_{t+1}}{F_t})-1$. The plots are highly revealing in showing that while there are large departures for S_{t+1} from F_t, the series itself is stationary as one would expect in an efficient market. In addition, the average value of the series is very low, that is, $\sum_{t=1}^{n}[(\frac{S_{t+1}}{F_t})-1]/n$ was -0.003 for the swiss franc-dollar, 0.001 for the pound-dollar, 0.174 for the yen dollar and -0.002 for the euro-dollar parity which are values very close to zero as one would expect in an efficient market.(6)

If the foreign exchange market is inefficient or contains a constant risk premium as empirical rejections of UIP seem to claim then the regression equation (22) is not well specified either. In this case, we can assume $E[S_{t+1}]=F_t+A$, where $A \neq 0$ is a constant risk premium(7). Then it follows from equation (14) that:

$$E[s_{t+1}] = \ln (F_t + A) - \frac{1}{2E[S_{t+1}]^2} Var(S_{t+1}). \tag{23}$$

If the deviation A from efficiency is small compared to the value of the forward contract the expected rate of depreciation can be expressed as:

$$E[\Delta s_{t+1}] = (f_t - s_t) + \frac{A}{F_t} - \frac{1}{2} Var(\frac{S_{t+1}}{F_t + A}) \tag{24}$$

given that $\ln(F_t + A) = \ln F_t + \ln(1+\frac{A}{F_t})$ and $\ln(1+\frac{A}{F_t}) \approx \frac{A}{F_t}$.

Under the same assumptions as before (negligible volatility of the ratio $S_{t+1}/(F_t+A)$) the UIP condition does not hold. The bias is the following:

$$E[\Delta s_{t+1}] - (f_t - s_t) = \frac{A}{F_t} \tag{25}$$

and the precise relationship between the relevant variables follows from equation (15):

$$\Delta s_{t+1} - (f_t - s_t) = \frac{A}{F_t} + [(\frac{S_{t+1}}{F_t + A}) - 1] \tag{26}$$

We have seen that UIP and market efficiency hold if condition (20) is satisfied, by using the equality given by equation (20) we obtain a regression model in the spirit of equation (7) and (22) to test for market inefficiency:

$$[(\frac{S_{t+1}}{F_t + A}) - 1] = \alpha - A\frac{1}{F_t} + \varepsilon_{t+1} \tag{27}$$

with α and A detecting any inefficiency and/or a risk premium in the foreign exchange market. In practice, however, this model is not tractable given that A is unknown and has an influence in the denominator of the dependent variable. However, equation (27) can be written as:

$$[(\frac{S_{t+1}}{F_t})\frac{1}{1+\frac{A}{F_t}} - 1] = \alpha - A\frac{1}{F_t} + \varepsilon_{t+1}, \tag{28}$$

and noting that $\frac{1}{1+\frac{A}{F_t}} = \sum_{i=0}^{\infty}(-\frac{A}{F_t})^i$ if $\left|\frac{A}{F_t}\right|$<1, the formula then reads as:

$$[(\frac{S_{t+1}}{F_t})\sum_{i=0}^{\infty}(-\frac{A}{F_t})^i - 1] = \alpha - \frac{A}{F_t} + \varepsilon_{t+1} \ . \tag{29}$$

If A is fairly small with respect to the forward exchange rate then $(\frac{A}{F_t})^2$ is negligible[8] and we can use a trimmed version of the previous model to test for foreign exchange

market efficiency that does not exhibit the pernicious statistical effects of omitting relevant variables in the regression equation. This model reads as:

$$[(\frac{S_{t+1}}{F_t})-1]+\frac{A}{F_t}-\frac{S_{t+1}A}{F_t^2}=\alpha+\varepsilon_{t+1} \ . \tag{30}$$

After some algebra, it can be shown that this model is equivalent to the following regression model:

$$[(\frac{S_{t+1}}{F_t})-1]=\frac{\alpha}{(1-\frac{A}{F_t})}+\frac{\varepsilon_{t+1}}{(1-\frac{A}{F_t})} \tag{31}$$

Using that $\frac{1}{1-\frac{A}{F_t}}=\sum_{i=0}^{\infty}(\frac{A}{F_t})^i$ if $\left|\frac{A}{F_t}\right|<1$ and denoting $\eta_{t+1}=\frac{\varepsilon_{t+1}}{(1-\frac{A}{F_t})}$ we have the following tractable regression model:

$$[(\frac{S_{t+1}}{F_t})-1]=\alpha+\alpha A\frac{1}{F_t}+\alpha A^2\frac{1}{F_t^2}+...+\alpha A^n\frac{1}{F_t^n}+\eta_{t+1} \tag{32}$$

with $V(\eta_{t+1})=\frac{\sigma_\varepsilon^2}{(1-\frac{A}{F_t})^2}$. Since we assume $(\frac{A}{F_t})^2$ and higher orders are negligible, this model can be simplified to:

$$[(\frac{S_{t+1}}{F_t})-1]=\alpha+\beta\frac{1}{F_t}+\eta_{t+1} \tag{33}$$

where α and $\beta=\alpha A$ measure the degree of market inefficiency. Although the variance of the error term is not homoskedastic the parameters estimates are unbiased and consistent. Note that once α and β are estimated and if the original sequence ε_{t+1} is homoskedastic one can carry out valid inference by using generalized ordinary least squares.

The third contribution of this section is a by-product of the previous analysis, it has been shown that even if the forward exchange rate is a biased predictor of the future spot exchange rate and this bias is constant, the model to be tested must make allowance for a time varying term given by $1/F_t$. Consequently, the parameter estimates of the standard formulations used to test UIP such as the regression equation (7) will be biased due to the omission of a relevant variable in the regressand. An appropriate regression equation devised to detect market inefficiencies of the type we have proposed should look at both the α and A terms in regression equations (32) or (33).

4. Empirical Evidence from the Foreign Exchange Market

We now proceed to analyse our proposed regressions for four major currencies against the US dollar; namely the swiss franc, yen, euro (Deutschmark) and the pound. The dataset consists of monthly exchange rate data and interest rate data covering the period November 1978 until January 2006 for these economies (n=327 observations). As can be seen in table 1, the tests of the conventional regression equation (7) are consistent with those of previous studies, such as Boothe and Longworth (1986), Cumby and Obstfeld (1984), Fama (1984) Maynard and Phillips (2001), Froot and Thaler (1990) in showing that uncovered interest rate parity is decisively rejected for all four parities. The slope of the regression equation is significantly negative for three of the four parities and negative for the euro-dollar parity. The estimated beta is shown to have values of -1 or less rather than the hypothesised value of unity, these decisive rejections of UIP have long been interpreted in the literature as indicative of foreign exchange market inefficiency and constitute the forward discount puzzle.

We now report regressions of equation (22) which are shown in table 2. The presence of possible heteroskedasticity in the error term is considered and modelled by a GARCH(1,1) model. The results are somewhat startling in that in contrast to the usual decisive rejections of UIP based on equation (7) our alternative test of UIP based on the right hand side of equation (21) and captured by the regression equation (22) shows that UIP appears to be holding and the foreign exchange market is efficient for all four parities studied. In addition, after filtering for conditional heteroskedastic volatility effects that are significant for the euro-dollar and pound-dollar parities the residuals exhibit characteristics of a white noise process as one would expect from a well behaved model.

As a further check, using the same data, we examine the results of our proposed regression test for market inefficiency as given by equation (33) which are reported in table 3. The reported results confirm those obtained by the regression equation (22), that is, we are unable to reject the null hypothesis of market efficiency. The estimated α is not significant in three of the four cases and the estimated β coefficient is likewise insignificant. We have to be careful when looking at the yen-dollar parity since the β parameter needs to be measured proportional to the exchange rate under study and, as such, it is no more significant than for the other parities. The analysis of the pound-dollar parity is less straightforward, in contrast to the results in table 2 both α and β parameters in table 3 are significant. However, the value of the intercept is offsets by the overall

contribution of the slope parameter divided by the forward rate yielding an almost negligible net effect [9].

The conclusions from our proposed tests are the complete reverse of those obtained from the conventional UIP tests. We argue that the negative beta's reported by equation (7) are highly misleading in suggesting the foreign exchange market is inefficient, our results suggest that the foreign exchange market has in fact been efficient for all four bilateral dollar parities under study. We argue that while the future spot exchange has deviated from the forward rate it has on average had no significant systematic bias and as such the foreign exchange market has been efficient.

5. Summary and conclusions

The conventional regression estimates of UIP taken at face value seem to indicate a significant degree of inefficiency in the foreign exchange market. The reported beta coefficients are usually negative and often significantly so. We have shown that the estimates of beta that have been obtained using conventional regression analysis are negatively biased and do not provide any indication of the degree of market inefficiency. This is particularly the case when the volatility of the dependent variable is high relative to the explanatory variable as is clearly the case in regressions of the change in the exchange rate relative to the forward discount/premium. In sum, we argue that the conventional regression estimates of UIP condition are seriously misspecified as tests of market efficiency.

In this study, by using the Taylor expansion of $\ln S_{t+1}$ about $E[S_{t+1}]$ we have derived the exact relationship between the difference of the log of spot exchange rate and the interest rate differential and shown that the conventional UIP condition does not hold in general unless deviations of S_{t+1} from F_t are *systematically* negligible. We have suggested two alternative tests of market efficiency one of which enables us to measure the degree of inefficiency of a foreign exchange market where it exists. Our estimates have shown that the foreign exchange market is efficient for all four bilateral dollar parities studied in stark contrast to the decisive rejections of market efficiency using the standard UIP regression. We believe that the UIP rejections of market efficiency are largely spurious due to the unbalanced nature of the regression and an inherent negative bias in the conventional regression. We argue that our proposed tests are able to distinguish between an efficient and an inefficient market more effectively than the conventional UIP regression. Our results indicate that there is no substantial evidence of *ex post* inefficiency in the foreign exchange market. In sum, our results are far more favourable to the concept of foreign exchange market efficiency and reveal that the forward discount puzzle, understood as a negative beta parameter, is primarily a statistical phenomenon. The results we report suggest that in addition to the vast amount of literature that has built on the forward discount puzzle, the large resulting offshoot literature on risk premia and rational/irrational bubbles will also need to be carefully re-evaluated.

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Footnotes

(1) The CIP formula is actually used by banks when quoting forward rates and it empirical relevance is well documented as in Sarno and Taylor (2002, chapter 2). For the purpose of this study we have calculated a set of forward rates from our interest and spot exchange rate data. This has the advantage of eliminating any timing mismatch between interest rates and exchange rates we use in the paper. The forward rates generated were extremely close to the actual forward rates we had data for. Using a set of actual forward exchange rates results in virtually identical results to those reported in this paper.

(2) The above equation needs to be modified according to whether one is looking at the one, three, six month or twelve month time horizon using the appropriate Eurocurrency interest rate to that time horizon. All the relevant adjustments have been made for the purposes of this paper.

(3) The ratio of the variance of the change in the exchange rate $(S_{t+1}-S_t)/S_t$ to the variance of the forward discount/premium $(F_{t+1}-S_t)/S_t$ for the period under study was 139.04 for the swiss franc-dollar, 223.40 for the pound-dollar, 223.63 for the yen-dollar and 162.67 for the euro-dollar parity.

(4) This is also known as the delta method.

(5) The average absolute values of $[(S_{t+1}/F_t)-1]$ and $[(S_{t+1}/F_t)-1]^2$ were for the swiss franc-dollar 0.028 and 0.001; 0.023 and 0.001 for the pound-dollar; 0.026 and 0.001 for the yen-dollar; and 0.025 and 0.001 for the euro-dollar.

(6) Note that the 0.174 for the yen-dollar rate when divided by the average spot exchange rate of 151.259 is 0.11%. which in absolute terms is lower than the -0.003 for the swiss franc divided by the average 1.601 spot exchange rate which is -0.19%.

(7) We use the term risk premium although it could be equally interpreted as an indicator of market inefficiency.

(8) Note that the nominal level of inefficiency A depends on the currency level. For the yen-dollar case, for example, a 1% inefficiency will be given by A=1.5 given an exchange rate of 150 yen/$. While for the pound-dollar a 1% inefficiency is given by 0.006 given an exchange rate of £0.60/$1.

(9) To see why this effect is negligible, the estimated beta coefficient is 0.023 and the average forward rate for the pound dollar parity was £0.610 so 1/average (F_t) – 1/0.61 =1.639 which means that 1.639 x 0.023 = 0.03769 which exactly offsets the -0.038 estimated alpha coefficient. This makes our reported results for the pound-dollar rate in table 3 consistent with those in table 2.

Data Sources:

Eurodollar, europound, euroyen euroswiss franc and eurodeutschmark/euroeuro interest and spot exchange rates from globalfindata.com for the period November 1978 until January 2006. Forward rates were generated from these data using the covered interest parity condition and were found to be extremely close to the set of actual forward rates from the same data source.

For the euro exchange rate we actually used the deutschmark parity up until January 1999 and then calculated an artificial deutschmark parity thereon using its fixed exchange rate against the euro. We refer to the euro exchange rate although it is technically a deutschmark pairty.

APPENDIX: TABLES AND FIGURES

Table 1. Estimated Equation $\Delta s_{t+1} = \alpha + \beta(f_t - s_t) + \varepsilon_{t+1}$

Currency pair	α	β	R^2	JB	ARCH
Swiss franc/dollar	-0.004	-1.46	0.015	3.75	0.66
	(0.067)	(0.02)		(0.15)	(0.57)
Yen/dollar	-0.008	-2.78	0.030	54.10	1.08
	(0.0001)	(0.0)		(0.00)	(0.50)
Euro/dollar	-0.001	-1.06	0.006	4.51	0.50
	(0.38)	(0.15)		(0.1)	(0.68)
Sterling/dollar	0.004	-2.60	0.030	87.6	6.10
	(0.03)	(0.001)		(0.0)	(0.0)

Note p-values for all of the tables are in parentheses

Table 2. Estimated Equation $[(\frac{S_{t+1}}{F_t})-1]=\alpha+\varepsilon_{t+1}$

Currency pair	α	R^2	JB	ARCH
Swiss franc/dollar	0.002	0.00	0.09	0.15
	(0.22)		(0.95)	(0.85)
Yen/dollar	0.001	0.00	13.9	0.07
	(0.39)		(0.00)	(0.93)
Euro/dollar	0.001*	0.00	4.77	0.83
	(0.39)		(0.09)	(0.43)
Sterling/dollar	-0.001*	0.00	1.36	0.40
	(0.47)		(0.50)	(0.66)

* Significant ARCH effects found when modelling conditional volatility of ε_{t+1}

Note p-values for all of the tables are in parentheses

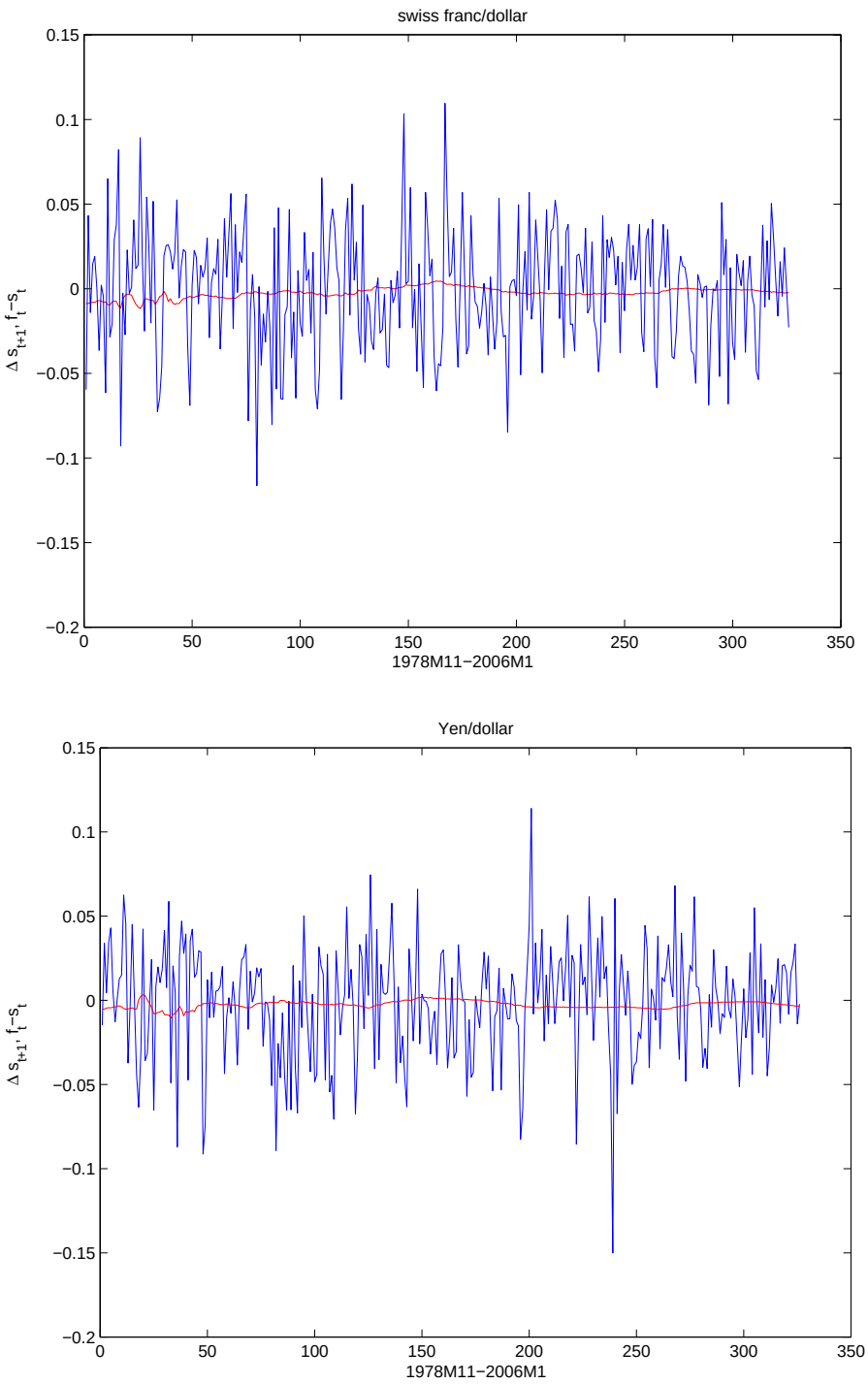
Table 3. Estimated Equation $[(\frac{S_{t+1}}{F_t})-1]=\alpha+\beta\frac{1}{F_t}+\eta_{t+1}$

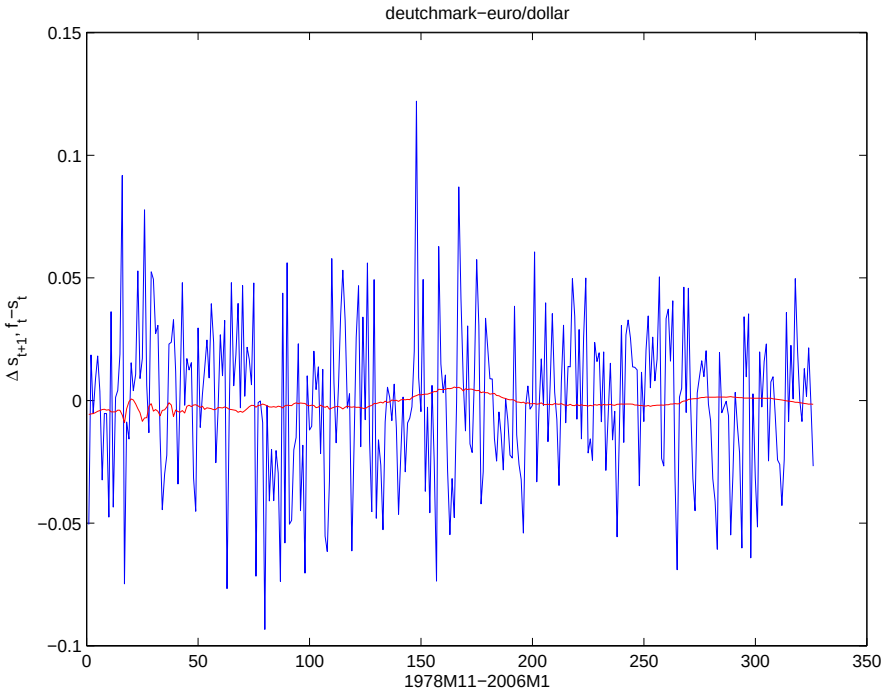
Currency pair	α	β	R^2	JB	ARCH
Swiss franc/dollar	-0.009	0.018	0.003	0.04	0.16
	(0.37)	(0.26)		(0.97)	(0.84)
Yen/dollar	-0.005	1.04	0.004	12.1	0.07
	(0.40)	(0.28)		(0.002)	(0.93)
Euro/dollar	-0.007*	0.016	0.002	3.83	0.42
	(0.45)	(0.36)		(0.14)	(0.85)
Sterling/dollar	-0.038*	0.023	0.009	1.29	0.01
	(0.001)	(0.001)		(0.52)	(0.67)

* Significant ARCH effects found when modelling conditional volatility of ε_{t+1}

Note p-values are in parentheses

Figure 1: Plot of Δs_{t+1} (blue) against $f_t - s_t$ (red) from equation (7) for monthly data of four different currencies: Swiss franc-dollar, Yen-dollar, Euro-dollar and Pound-dollar for the period November 1978-January 2006.




deutchmark−euro/dollar
0.15
0.1
0.05
0
−0.05
−0.1
0
50
100
150
200
250
300
350
1978M11−2006M1
Δ s_{t+1}, f_t−s_t

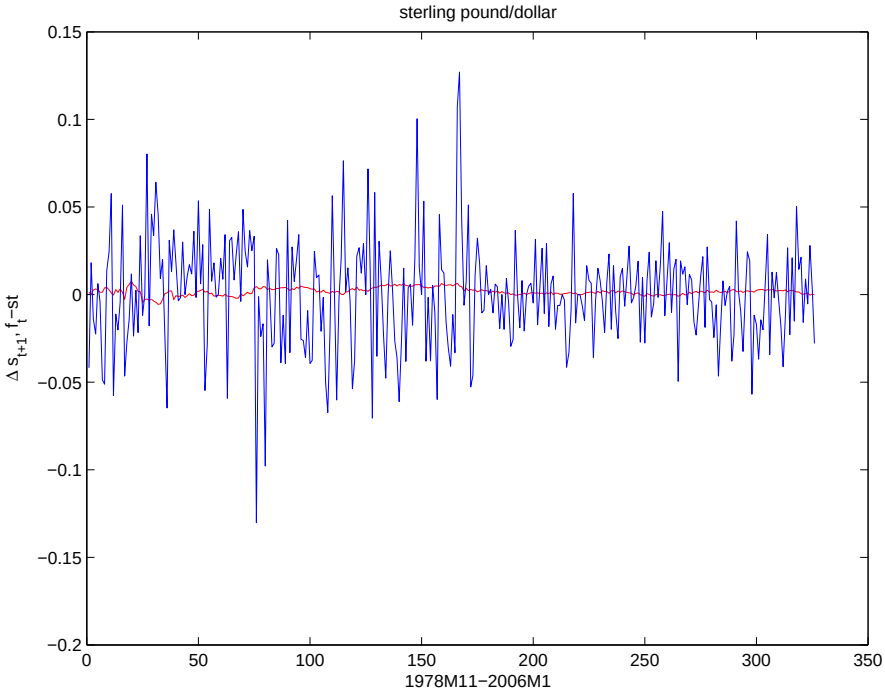

sterling pound/dollar
0.15
0.1
0.05
0
−0.05
−0.1
−0.15
−0.2
0
50
100
150
200
250
300
350
1978M11−2006M1
Δ s_{t+1}, f_t−st

Figure 2: Upper panels depict $(\frac{S_{t+1}}{F_t})-1$ and lower panels the term $-0.5[(\frac{S_{t+1}}{F_t})-1]^2$ for monthly data of four different currencies: Swiss franc-dollar, Yen-dollar, Euro-dollar and Pound-dollar for the period November 1978-January 2006.

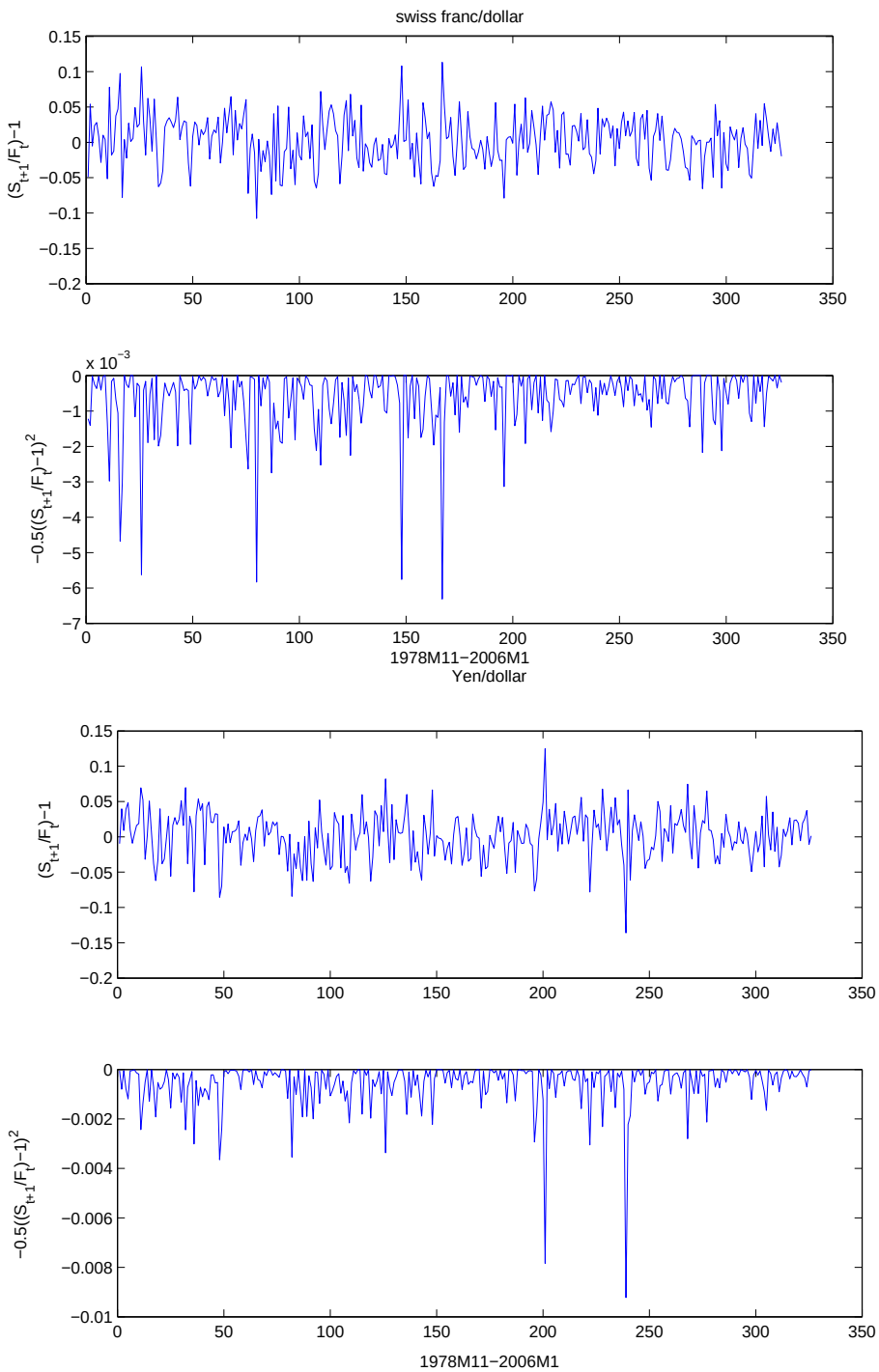


Figure 2. (Continued)