Non-linear Control Theory and Applications in Power and Energy Systems

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Outline

- Introduction
- Model of some typical Power & Energy Systems
- Non-linear control theory
  - Dissipative Hamiltonian form
  - Oscillator-based non-linear controller
  - Bounded integral controller (BIC)
- Applications
  - DC/DC boost converter (energy storage, PV)
  - Wind power
  - Parallel operation of inverters
  - Grid-friendly inverters (Synchronverters): CAPS
  - Rectifiers with limited current
- Summary
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- Summary
Power systems

- Increase of renewable energy systems penetration
  - Connection to the electrical grid through power electronic devices (converters)
- Power grid stability becomes fragile!
- Conventional control techniques and linear systems analysis → not enough
Control in Power Systems

Power electronics have high efficiency. Power Grid stability lies on the system operation and control.

The need to create a bridge between 'automatic control' and 'power systems' communities

- Accurate modelling of power systems
- Advanced control design
- Passivity - Stability

Non-linear control theory is essential!
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Non-linear control theory is essential!
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- **Summary**
DC/DC boost converter

Applications: PV, energy storage, wind, etc.

Aim: Regulation of the dc output voltage $v$ to a higher level than the dc input voltage $E$ by controlling the switch $u$. 

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Average model

Average modelling \((u \rightarrow \mu)\)

- high switching frequency
- duty-ratio \(\mu = \frac{t_{on}}{T}\)

\[
\begin{align*}
L \dot{i} &= -(1 - \mu)v + E \\
C \dot{v} &= (1 - \mu)i - \frac{1}{R}v
\end{align*}
\]

where the control input \(\mu\) is continuous-time and \(\mu \in [0,1]\).

The system is non-linear!
AC/DC or DC/AC converter

Average modelling

+ Park transformation \((a - b - c \rightarrow d - q)\)

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Average model

\[ L \dot{i}_d = -R i_d + \omega_s L i_d - m_d \frac{V_{dc}^2}{2} + U_d \]

\[ L \dot{i}_q = -R i_q - \omega_s L i_q - m_q \frac{V_{dc}^2}{2} + U_q \]

\[ C \dot{V}_{dc} = \frac{3}{4} (m_d i_d + m_q i_q) - \frac{V_{dc}}{R_L} \]

- external uncontrolled inputs: grid voltages \( U_d, U_q \)
  (constants, usually \( U_q = 0 \))
- control inputs: duty-ratio signals \( m_d = \frac{2V_d}{V_{dc}}, \; m_q = \frac{2V_q}{V_{dc}} \)

In order to operate the converter in the 'linear modulation' (PWM):

\[ m_d^2 + m_q^2 \leq 1 \]
Existing control techniques

Traditional techniques:
- linearisation (small-signal model)
- linear control (PI)

Advanced techniques:
- non-linear techniques (passivity-based, feedback linearisation) → asymptotic stability
- dependence from system parameters

Can we find a non-linear parameter-free controller with guaranteed closed-loop stability?
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Summary
Generalised dissipative Hamiltonian form

DC/DC boost converter:

\[ L \dot{i} = -(1 - \mu) v + E \]
\[ C \dot{v} = (1 - \mu) i - \frac{1}{R} v \]

\[ \rightarrow M \dot{x} = (J(\mu) - R) x + Gu \]

where \( x = [ i \ v ]^T \), \( \mu \) is the control input and \( u = E \) is the external uncontrolled input.

\[ M = \begin{bmatrix} L & 0 \\ 0 & C \end{bmatrix}, \quad J(\mu) = \begin{bmatrix} 0 & -(1-\mu) \\ (1-\mu) & 0 \end{bmatrix}, \quad R = \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{R} \end{bmatrix}, \quad G = [1 \ 0]^T \]

The system is passive w.r.t. the external uncontrolled input \( u \) independently from \( \mu \rightarrow \text{Dynamic controller?} \)
Non-linear control design

\[ M \dot{x} = (J(x, \mu) - R)x + G(x)u \]

- Passive w.r.t. the constant uncontrolled input \( u \)
- Control input \( \mu \) is a saturated signal in the interval \((-1,1)\) or \([0,1)\)

**Control task:** Regulate state \( x_i \) at \( x_i^{\text{ref}} \).

Oscillator-based non-linear controller:

\[
\begin{align*}
\mu &= z_1 + c \\
\begin{bmatrix}
\dot{z}_1 \\
\dot{z}_2
\end{bmatrix} &=
\begin{bmatrix}
0 & -k(x_i - x_i^{\text{ref}}) \\
k(x_i - x_i^{\text{ref}}) & 0
\end{bmatrix}
\begin{bmatrix}
z_1 \\
z_2
\end{bmatrix}, \quad k > 0
\end{align*}
\]

- \( \mu \in (-1,1) \): \( c = 0 \), \( z_1(0) = -1 + \gamma \), \( z_2(0) = 0 \)
- \( \mu \in [0,1) \): \( c = \frac{1-\gamma}{2} \), \( z_1(0) = -\frac{1-\gamma}{2} \), \( z_2(0) = 0 \)
Non-linear control design

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k(x_i - x_{i}^{\text{ref}}) & 0
\end{bmatrix}
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\end{bmatrix}, \quad k > 0
\]

- \( \mu \in (-1, 1): \ c = 0, \ z_1(0) = -1 + \gamma, \ z_2(0) = 0 \)
- \( \mu \in [0, 1): \ c = \frac{1-\gamma}{2}, \ z_1(0) = -\frac{1-\gamma}{2}, \ z_2(0) = 0 \)
Controller operation

$\rightarrow$ the control input $\mu$ is bounded in $[0, 1 - \gamma]$!
Closed-loop system stability

Closed-loop system:

\[ \tilde{M} \dot{\tilde{x}} = \left( \tilde{J}(\tilde{x}) - \tilde{R} \right) \tilde{x} + \tilde{G} u \]

with \( \tilde{x} = \begin{bmatrix} x^T & z_1 & z_2 \end{bmatrix}^T \).

\[ \tilde{M} = \begin{bmatrix} M & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \tilde{R} = \begin{bmatrix} R & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \tilde{G} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T, \]

\[ \tilde{J}(\tilde{x}) = \begin{bmatrix} J & 0 & 0 \\ 0 & 0 & -k(x_i - x_i^{ref}) \\ 0 & k(x_i - x_i^{ref}) & 0 \end{bmatrix} \]

We keep the same passive structure!
Closed-loop system stability

The converter system and the controller system can be handled independently:

- converter system with $\mu \in [0, 1 - \gamma]$ is bounded input-bounded output (BIBO) stable.
- controller system is BIBO with zero gain!
  (bounded output independently from the input)
→ closed-loop system is BIBO w.r.t. the external input $u$.
Since $u$ is constant, then the closed-loop system solution is bounded in an area where the desired equilibrium exists!
  Unique equilibrium, no limit cycles → convergence to the equilibrium!
Bounded integral controller (BIC)

Traditional Integral controller for regulating a scalar function $g(x)$ to zero:

$$u(t) = \int_0^t g(x(\tau)) \, d\tau$$

which introduces a dynamic controller that can be written as

$$\begin{align*}
  u &= w \\
  \dot{w} &= g(x)
\end{align*}$$

Bounded Integral Controller (BIC):

$$\begin{bmatrix}
  \dot{w} \\
  \dot{w}_q
\end{bmatrix} = \begin{bmatrix}
  -k \left( \frac{w^2}{u_{\text{max}}^2} + \frac{(w_q-b)^2}{\varepsilon^2} \right) - 1 & g(x)c \\
  -\frac{\varepsilon^2}{u_{\text{max}}^2} g(x)c & -k \left( \frac{w^2}{u_{\text{max}}^2} + \frac{(w_q-b)^2}{\varepsilon^2} \right) - 1
\end{bmatrix} \begin{bmatrix}
  w \\
  w_q
\end{bmatrix}$$

$b \geq 0$, $k$, $u_{\text{max}}$, $\varepsilon$, $c > 0$. 

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Consider a non-linear system: \( \dot{x} = f(t, x, u, u_1) \)

**Proposition**

The feedback interconnection of plant system with the proposed BIC is ISpS, when the plant system is ISpS.

**ISpS:** Input-to-state practical stability

The BIC introduces a zero-gain property.
BIC with given bound $u \in [-u_{\text{max}}, u_{\text{max}}]$

- Normal conditions: $b = 0$, $\varepsilon = 1$, $k > 0$, $c = \frac{w_q u_{\text{max}}^2}{u_{\text{max}}^2 - (u^*)^2}$ where $u^*$ is the nominal value of $u$.
- Abnormal conditions: $b = c = 1$, $k > 0$ (sufficiently large), $\varepsilon > 0$ (sufficiently small)
Integrator replacement

Bounded integral control (BIC):

normal conditions: slow down the integration near the bounds
abnormal conditions: fail-safe operation
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Summary
Control task: Regulate the output voltage $v$ at $v_{ref}$.
Using the oscillator-based non-linear controller:

$$\mu = z_1 + \frac{1 - \gamma}{2}$$

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} 0 & -k(v - v_{ref}) \\ k(v - v_{ref}) & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

- $\gamma$ small positive constant
- $k > 0$
- initial conditions $z_1(0) = -\frac{1 - \gamma}{2}$, $z_2(0) = 0$
Results

![Image of experimental setup with LabVIEW software and hardware components]

Graph showing time (sec) on the x-axis and output voltage (V) on the y-axis, with comparison between simulation and experiment.

Graph showing complex plane with points labeled A, B, Wμ(0), and Z1, Z2 coordinates.

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**Generator-side converter control:**
- Maximum Power Point Tracking (MPPT)
- Field-oriented control (FOC) of SCIG

**Grid-side converter control:**
- dc-link bus voltage regulation
- unity power factor \( \left( Q_g = 0 \right) \)

Traditional control techniques: PI and cascaded PI control (complete system stability has not been investigated yet)
Generator-side converter control:
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Complete wind power system modelling: SCIG dynamics + ac/dc/ac converter dynamics

\[ M \dot{x} = (J(x, m_{ds}, m_{qs}, m_{dg}, m_{qg}) - R)x + u \]

where \( x = [i_{ds}, i_{qs}, \lambda_{dr}, \lambda_{qr}, \omega_r, i_d, i_q, V_{dc}]^T \) is the state vector, \( u = [0, V_m, 0, 0, -\frac{2}{3}T_m, 0, 0, 0]^T \) is the uncontrolled external input vector.

- Generator-side converter control inputs: \( m_{ds}, m_{qs} \)
- Grid-side converter control inputs: \( m_{dg}, m_{qg} \)
Wind power system modelling

\[ M = \text{diag} \left( \sigma, \sigma, \frac{1}{L_r}, \frac{1}{L_r}, \frac{2}{3} J_m, L_g, L_g, \frac{2}{3} C \right), \]

\[
 R = \begin{bmatrix}
 \frac{R_r L^2_m}{L_f^2} + R_s & 0 & -\frac{R_r L_m}{L_f^2} & 0 & 0 & 0 & 0 & 0 \\
 0 & \frac{R_r L^2_m}{L_f^2} + R_s & 0 & -\frac{R_r L_m}{L_f^2} & 0 & 0 & 0 & 0 \\
 -\frac{R_r L_m}{L_f^2} & 0 & \frac{R_r L^2_m}{L_f^2} & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & \frac{R_r L^2_m}{L_f^2} & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & \frac{2}{3} b & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & R_g & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & R_g & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{2 R_{dc}}{3} \\
 \end{bmatrix}
\]

\[
 J = \begin{bmatrix}
 0 & \sigma \omega_e & 0 & 0 & 0 & \frac{L_m}{L_r} p \lambda_{qr} & 0 & 0 & \frac{1}{2} m_{ds} \\
 -\sigma \omega_e & 0 & 0 & 0 & \frac{L_m}{L_r} p \lambda_{dr} & 0 & 0 & \frac{1}{2} m_{qs} \\
 0 & 0 & 0 & \omega_e - p \omega_r & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & -p \omega_r & 0 & 0 & 0 & 0 \\
 -\frac{L_m}{L_r} p \lambda_{qr} & \frac{L_m}{L_r} p \lambda_{dr} & 0 & 0 & 0 & 0 & \omega_s L_g & -\frac{1}{2} m_{dg} \\
 0 & 0 & 0 & 0 & 0 & -\omega_s L_g & 0 & -\frac{1}{2} m_{qg} & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} m_{dg} & \frac{1}{2} m_{qg} & 0 \\
 -\frac{1}{2} m_{ds} & -\frac{1}{2} m_{qs} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 \end{bmatrix}
\]
How can we control the complete wind power system to achieve the desired operation and guarantee stability?

→ use the same oscillator-based non-linear controller for the duty-ratio inputs as in the case of the dc/dc boost converter

The limits of the duty-ratio signals are not independent:

\[
m_{ds}^2 + m_{qs}^2 \leq 1, \quad m_{dg}^2 + m_{qg}^2 \leq 1
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Generator-side converter control:

- **MPPT**: regulate rotor speed $\omega_r$ to $\omega_r^{\text{ref}}$
- **FOC**: regulate $d-$axis stator current $i_{ds}$ to $i_{ds}^{\text{ref}}$

\[
\begin{align*}
    m_{ds} &= z_1 \\
    m_{qs} &= z_2 \\
\end{align*}
\]

\[
\begin{bmatrix}
    \dot{z}_1 \\
    \dot{z}_2 \\
    \dot{z}_3 \\
\end{bmatrix} =
\begin{bmatrix}
    0 & 0 & -k_1 (i_{ds} - i_{ds}^{\text{ref}}) \\
    0 & 0 & -k_2 (\omega_r - \omega_r^{\text{ref}}) \\
    k_1 (i_{ds} - i_{ds}^{\text{ref}}) & k_2 (\omega_r - \omega_r^{\text{ref}}) & -c_1 (z_1^2 + z_2^2 + z_3^2 - 1) \\
\end{bmatrix}
\begin{bmatrix}
    z_1 \\
    z_2 \\
    z_3 \\
\end{bmatrix}
\]

$k_1, k_2$ are non-negative constants
The 3 controller states are attracted and exclusively move on the surface of the sphere \( C_{r1} = \{ z_1, z_2, z_3 : z_1^2 + z_2^2 + z_3^2 = 1 \} \).

The duty-ratio inputs \( m_{ds} = z_1 \) and \( m_{qs} = z_2 \) take values inside the disk \( D \), i.e. \( m_{ds}^2 + m_{qs}^2 \leq 1 \).
Real-time simulation results

- $\omega_r: [0.2 \text{pu/div}]$
- $\omega_r^{ref}: [0.2 \text{pu/div}]$
- $i_{ds}: [0.5 \text{pu/div}]$
- $i_d: [0.5 \text{pu/div}]$
- $i_{qs}: [0.5 \text{pu/div}]$
- $i_q: [0.5 \text{pu/div}]$
- $V_{dc}: [0.25 \text{pu/div}]$
- $Q_g: [0.5 \text{pu/div}]$
- $P_g: [0.5 \text{pu/div}]$
- $P_{\text{wind}}: [0.5 \text{pu/div}]$
- $V_{\text{out}}: [1 \text{pu/div}]$
- $i_{\text{out}}: [1 \text{pu/div}]$

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Non-linear Control in Power Systems
A power network has a large number of buses:

The inverters can be assumed as voltage sources to simplify the analysis.
Is non-linear control theory important in this case?

- Basic control structure: Droop control (non-linear structure of $P$ and $Q$)
- Accurate load modelling (linear, non-linear)

→ Linearisation and local stability is not enough
Is non-linear control theory important in this case?

- Basic control structure: Droop control (non-linear structure of $P$ and $Q$)
- Accurate load modelling (linear, non-linear)

→ Linearisation and local stability is not enough
Parallel operation of inverters

Proportional load sharing:

- Conventional droop control ($i \in \{1, 2\}$):
  
  \[
  E_i = E^* - n_i Q_i \\
  \dot{\theta}_i = \omega^* - m_i P_i \\
  \Rightarrow \quad v_{ri} = \sqrt{2}E_i \sin(\theta_i)
  \]

  cannot achieve accurate load sharing when inverters introduce different output impedances.
Robust Droop Controller (RDC):

\[
\dot{E}_i = K_e (E^* - V_o) - n_i Q_i
\]

\[
\dot{\theta}_i = \omega^* - m_i P_i
\]

- accurate load sharing
- output voltage regulation near $E^*$

Linear/non-linear plant + non-linear controller

Is the closed-loop system stable?
Robust Droop Controller (RDC):

\[ \dot{E}_i = K_e (E^* - V_o) - n_i Q_i \]

\[ \dot{\theta}_i = \omega^* - m_i P_i \]

- accurate load sharing
- output voltage regulation near \( E^* \)

Linear/non-linear plant + non-linear controller

\[ \Rightarrow \text{Is the closed-loop system stable?} \]
Bounded droop controller (BDC):

RMS dynamics:

\[
\begin{align*}
\dot{E}_i &= (K_e (E^* - V_o) - n_i Q_i) c E_{qi} \\
\dot{E}_{qi} &= - (K_e (E^* - V_o) - n_i Q_i) c E_i
\end{align*}
\]

where \( c \) is a positive constant.

frequency dynamics:

\[
\begin{align*}
\dot{z}_i &= (\omega^* - m_i P_i) z_{qi} \\
\dot{z}_{qi} &= - (\omega^* - m_i P_i) z_i.
\end{align*}
\]

\[\Rightarrow v_{ri} = \sqrt{2E_i z_i}\]
**BDC operation**

**RMS voltage dynamics**

\[ \dot{\phi}_i = (K_e (E^* - V_o) - n_i Q_i) c \]

\[ E_i, E_{qi} \in [-V_i, V_i] \]

Therefore \( v_{ri} = \sqrt{2}E_iz_i \in [-\sqrt{2}V_i, \sqrt{2}V_i] \Rightarrow BDC \)

**frequency dynamics**

\[ \dot{\theta}_i = \omega^* - m_i P_i \]

\[ z_i, z_{qi} \in [-1, 1] \]
Real-time simulations (non-linear load)

BDC

RDC

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Non-linear Control in Power Systems
Real-time simulations (non-linear load)

BDC

$E_{q1} : [200V/div]$

$E_1 : [200V/div]$

$z_{q1} : [1/div]$

$z_1 : [1/div]$

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Grid-friendly inverters (Synchronverters)

Rotor field axis

Rotation

Field voltage

(a-axis)

$(\theta = 0)$

Field voltage

Rotation

$b$-axis

$c$-axis

$i_b$

$i_c$

$i_a$

$V_a$

$V_f$

$V_b$

$V_c$

$R_s, L$

$R_f, L_f$

$M$

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Synchronverter parts

Power Part

Control Part

Formulas of $T_e$, $Q$, $e$

$P_{set} \frac{p}{\dot{\theta}_n} T_m$ $\frac{1}{J_s} \frac{1}{s}$ $\omega_r$ $\omega$ $\theta_r$ $\theta$ $\theta_e$ $e$ $i$ $V_e$

PWM generation

From to the power part

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Stability is still not proven!

Can we guarantee system stability while maintaining the synchronverter original operation?

Solution: Use BIC for the integrators of the frequency $\omega$ and the field-excitation current $i_f$ loops.

- Guarantee specific bounds for the synchronverter voltage and frequency according to its technical limits:

  $$E \in [E_{\text{min}}, E_{\text{max}}] \quad \text{and} \quad \omega \in [\omega_n - \Delta \omega_{\text{max}}, \omega_n + \Delta \omega_{\text{max}}]$$

- Guarantee convergence to the desired unique equilibrium
Synchronverter control and stability

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  $$E \in [E_{\text{min}}, E_{\text{max}}] \quad \text{and} \quad \omega \in [\omega_n - \Delta \omega_{\text{max}}, \omega_n + \Delta \omega_{\text{max}}]$$

- Guarantee convergence to the desired unique equilibrium
Real-time simulation results

Ps: [50W/div]
Qs: [500Var/div]
Time: [5s/div]

f-fn: [0.1Hz/div]
fg - fn: [0.1Hz/div]

E: [0.5pu/div]
V-Vg: [0.5V/div]
Time: [5s/div]

va: [10V/div]
i_ga: [10A/div]
Time: [20ms/div]
Big step for guaranteeing stability of CAPS!
Closed-loop system stability

A fundamental problem in control systems is that closed-loop system stability is not guaranteed even if the plant is BIBO.

The BIC can guarantee a bounded input for the plant independently from the error signal (zero-gain property).

However, a given bound for a state of the plant (e.g. current) is not guaranteed.

Can we design such a controller?
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Can we design such a controller?
Single-phase rectifier with limited current

\[ L \frac{di}{dt} = -ri - v + v_s \]

\[ CV_{dc} \frac{dV_{dc}}{dt} = vi - \frac{V_{dc}^2}{R_L} \]

Control tasks:
- dc output voltage regulation at \( V_{dc}^{\text{ref}} \)
- unity power factor operation (ac side)

Control input: converter voltage \( v \)
Traditional control of rectifiers

- Outer-loop PI controller for voltage regulation
- Inner-loop Hysteresis controller for current control

→ stability and a given limit for the input current are not guaranteed, requires a PLL

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Current limiting non-linear controller (CLNC):

\[
\begin{align*}
    v(t) &= w(t)i(t) \\
    \begin{bmatrix}
    \dot{w} \\
    \dot{w}_q
    \end{bmatrix} &= \begin{bmatrix}
    0 & c w_q (V_{dc} - V_{dc}^{ref}) \\
    -\frac{c w_q}{\Delta w_{max}^2} (V_{dc} - V_{dc}^{ref}) & -k \left( \frac{(w - w_m)^2}{\Delta w_{max}^2} + w_q^2 - 1 \right)
    \end{bmatrix} \begin{bmatrix}
    w - w_m \\
    w_q
    \end{bmatrix}
\end{align*}
\]

with \( c, k, w_m, \Delta w_{max} > 0 \).

If the output voltage \( V_{dc} \) is regulated at \( V_{dc}^{ref} \), then \( w = w^* \) and

\[
v(t) = w^* i(t)
\]

which guarantees unity power factor at the input of the rectifier.
Experimental results

Normal operation: $V_{dc}^{\text{ref}} = 90\,\text{V} \rightarrow 115\,\text{V}$

<table>
<thead>
<tr>
<th>Operation</th>
<th>$U_{\text{rms}}$</th>
<th>$I_{\text{rms}}$</th>
<th>$V_{\text{L}}$</th>
<th>$I_{\text{L}}$</th>
<th>$I_{\text{dc}}$</th>
<th>$V_{\text{dc}}$</th>
<th>Current Limit</th>
<th>$I_{\text{max}} = 4,\text{A}$: $V_{dc}^{\text{ref}} = 90,\text{V} \rightarrow 140,\text{V}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal</td>
<td>27.112</td>
<td>3.137</td>
<td>85.05</td>
<td>83.38</td>
<td>26.614</td>
<td>26.614</td>
<td>Current Limit</td>
<td>$I_{\text{max}} = 4,\text{A}$: $V_{dc}^{\text{ref}} = 90,\text{V} \rightarrow 140,\text{V}$</td>
</tr>
<tr>
<td>Current</td>
<td>16.74 var</td>
<td>0.9804</td>
<td>114.99 var</td>
<td>123.96 var</td>
<td>16.30 var</td>
<td>16.30 var</td>
<td>Current Limit</td>
<td>$I_{\text{max}} = 4,\text{A}$: $V_{dc}^{\text{ref}} = 90,\text{V} \rightarrow 140,\text{V}$</td>
</tr>
<tr>
<td>Limit</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td>$I_{\text{max}} = 4,\text{A}$: $V_{dc}^{\text{ref}} = 90,\text{V} \rightarrow 140,\text{V}$</td>
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Outline

- Introduction
- Model of some typical Power & Energy Systems
- Non-linear control theory
  - Dissipative Hamiltonian form
  - Oscillator-based non-linear controller
  - Bounded integral controller (BIC)
- Applications
  - DC/DC boost converter (energy storage, PV)
  - Wind power
  - Parallel operation of inverters
  - Grid-friendly inverters (Synchronverters): CAPS
  - Rectifiers with limited current
- Summary
Stability and advanced operation of power systems require:

- accurate system modelling (average analysis → dissipative Hamiltonian structure)
- non-linear control design (parameter-free)
  - Oscillator-based non-linear controller
  - Bounded integral controller

Non-linear system analysis

- Passivity analysis
- zero-gain property of the controller
- convergence to the desired equilibrium
Control applications

- Power converters
- Renewable energy systems (wind, solar, etc.)
- Parallel operation of inverters
- Grid-friendly inverters (Synchronverters)
- many more...

Non-linear control theory can change the way power systems have been treated so far!
Control applications

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