**Introduction: Examples**

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- **Social Networks**: Telecommunication company selection, Opinion about an idea or a product, Selection of fashion group, Engagement in criminal behavior, etc...

- **Non-Social Networks**: Stores for renting, Gas Station Prices, etc...
Game Description: Dynamics and Costs

**Interaction Structure:**
- Set of players \((p_1, \ldots, p_N)\)
- Each player \(p_i\) has a type \(\theta_i \in \Theta\).
- Graph of interactions \(G\).

**Dynamics:**
\[
x_{i, k+1} = \sum_{j \in N_i^1(G)} f_{\theta_i, \theta_j}(x_{i, k}, x_{j, k}, u_{i, k}, u_{j, k}, w_{i, k})
\]

**Cost Functions:**
\[
J_i = E \left\{ \sum_{k=0}^{T} \rho^k \left[ \sum_{j \in N_i^1(G)} g_{\theta_i, \theta_j}(x_{i, k}, x_{j, k}, u_{i, k}, u_{j, k}) \right] \right\}
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**Information:**
- **Local**
- **Statistical:** The players consider a statistical ensemble of games
Definition of an approximate equilibrium concept for ensembles of games.

**Definition (ε - Probabilistic Approximate Nash Equilibrium)**

Consider an ensemble of Interaction Structures $\mathcal{E}$. A set of strategies $(\gamma_i)_{i=1}^N$ is ε - Probabilistic Approximate Nash Equilibrium for the ensemble $\mathcal{E}$ if it holds:

$$P\left(\{S \in \mathcal{E} : (\gamma_i)_{i=1}^N \text{ is an } \varepsilon - \text{Nash equilibrium}\} \right) > 1 - \varepsilon$$
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- How much information is needed to have a PAN equilibrium.

**Complexity Functions**
Approximate Equilibrium: Statistical Physics Analog

The total energy in the canonical ensemble

I. Kordonis, G. P. Papavassilopoulos... meeting
Special Cases

Examples of games with low complexity:

- Static or LQ games on *Erdos-Reyni* random graphs or *Small World* nets: **Law of large numbers**
- Static game on *Lattices*: **Contraction mapping ideas**
- LQ game on a *Ring*: **Low gain to distant players**
- A non-quadratic static game on a *ring*: **Cooperation among the players**
Dynamic Rules

What if there is not enough information?

- Nash Equilibrium
  - Stochastic Adaptive Control problem (Dual Control Problem): The actions of each player affect her own future estimation
  - The actions of each player affect the future estimation of the other players (like the Witsenhausen’s counterexample)
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- **Dynamic/Adaptation Rules:**
  - Simple rules though non-optimal, would probably lead to Nash equilibrium asymptotically
  - Bounded Rationality
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**Examples:**
- Adaptive Control Laws
- Learning
- Best Response
- Evolutionary Dynamics
The Cheating Problem

- **The Cheating Problem**: The Adaptation rule of a player may be used by the others in order to manipulate her.
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Questions

1. When can a Dynamic/Adaptation rule serve as a prediction of the behavior of the players? Assessment
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Questions

1. When can a Dynamic/Adaptation rule serve as a prediction of the behavior of the players? **Assessment**
2. Are there any simple cheating strategies?
3. What are the outcomes of the game when (all or some of) the participants are cheating?
Simple cheating strategy: **Pretending**

1. Game outcomes **alternative** to the Nash equilibrium.
2. Interesting relationships between **pretending** and **leadership**
3. Pretending may enhance **cooperation**, **competition** or even make a system designed to work well on the Nash equilibrium **not working at all**
Future Directions

- Time varying network topologies
- Develop testable conditions to assess the adaptive laws and develop laws less sensitive to cheating
- Network design to reduce the ability to cheat